MULTIPLE CHOICE, 5 Points Each

1 Consider the following sets:

$$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$

$$A = \{1, 3, 5, 7, 9\}$$

$$B = \{2, 4, 6, 8, 10\}$$

$$C = \{1, 2, 3, 4, 5\}.$$

Which of the following is the set $(A \cap C) \cup B$?

(a) $\{1, 2, 3, 4, 5\}$ $\underline{(b)}$ $\{1, 2, 3, 4, 5, 6, 8, 10\}$ (c) $\{1, 2, 3, 4, 5, 7, 9\}$

(d) $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ (e) $\{1, 2, 3, 4, 5, 6, 7\}$

2 A survey of 100 coffee drinkers showed that 70 take sugar, 60 take cream and 50 take both sugar and cream with their coffee. How many of the coffee drinkers surveyed take neither sugar nor cream with their coffee?

(a) 50 (b) 30 $\underline{(c)}$ 20 (d) 10 (e) 15

3 A survey of 100 people using the reading room in the library revealed the following results:

40 read Time magazine

30 read The New York Times

25 read The Irish Times

15 read Time and The New York Times

12 read Time and The Irish Times

10 read the New York Times and The Irish Times

4 read all three

How many of the people surveyed read at least one of the three newspapers?

(a) 75 (b) 38 (c) 95 (d) 25 <u>(e)</u> 62

4 Piraullis pizza joint offers a mix and match pizza on its menu. There are 4 different meats to choose from, 5 different vegetables, 4 different types of cheese, and 2 different types of crust. How many different types of Pizza can be made by choosing 1 type of meat, 1 vegetable, 1 cheese and 1 crust?

(a) 80 (b) 4 (c) 20 <u>(d)</u> 160 (e) 49

5 A company has ten cars and ten garages. Each night each car is placed in a garage - one car to one garage. In how many ways can the cars be put away?

<u>(a)</u> $10!$ (b) 2^{10} (c) $C(10,1)$ (d) $P(10,5)$ (e) 10^2

6 An Urn contains 5 numbered blue marbles and 10 numbered red marbles. How many samples of 4 marbles can be drawn from the urn with exactly 2 red marbles and 2 blue marbles.

(a) $2^5 2^{10}$ (b) $C(15,2)$ (c) $P(5,2)P(10,2)$ (d) $5^2 10^2$ <u>(e)</u> $C(5,2)C(10,2)$

7 If an unbiased coin is flipped 8 times and the sequence of heads and tails is recorded. What is the probability that the sequence will contain exactly 3 heads?

(a) $\frac{P(8,3)}{2^8}$ (b) $\frac{3}{2^8}$ $\underline{(c)}$ $\frac{C(8,3)}{2^8}$ (d) $\frac{3}{8}$ (e) $1 - \frac{C(8,5)}{2^8}$

8 A die is rolled twice and the numbers on the uppermost faces are recorded. Consider the events:

E : At least one number is odd
F : Both numbers are even
G : exactly one of the numbers is 2

Which of the following are a pair of mutually exclusive events?

(a) F and G (b) E and F' (c) E and G <u>(d)</u> E and F (e) E and G'

9 In a certain experiment, the events E and F satisfy $Pr(E) = 0.5$, $Pr(F) = 0.6$ and $Pr(E \cup F) = 0.9$, what is $Pr(E|F)$?

(a) $\frac{3}{5}$ (b) $\frac{2}{3}$ (c) $\frac{2}{5}$ $\underline{(d)}$ $\frac{1}{3}$ (e) $\frac{1}{5}$

10 Which of the following tables serve as a probability distribution for an experiment with the sample space $\{s_1, s_2, s_3, s_4, s_5\}$.

(a)

k	Pr($X = k$)
s_1	.3
s_2	.2
s_3	.2
s_4	.1
s_5	.4

(b)

k	Pr($X = k$)
s_1	.5
s_2	.1
s_3	.2
s_4	.2
s_5	.3

(c)

k	Pr($X = k$)
s_1	.2
s_2	.3
s_3	.1
s_4	.1
s_5	.4

(d)

k	Pr($X = k$)
s_1	1
s_2	0
s_3	.2
s_4	1.2
s_5	.3

$\underline{(e)}$

k	Pr($X = k$)
s_1	.3
s_2	.1
s_3	.2
s_4	.1
s_5	.3

11 The quality control inspector at the Enlightenment lightbulb factory is about to inspect a box of 10 lightbulbs, three of which are defective. He will take a bulb from the box and test it. If the bulb does not work, the box will not be shipped. If the bulb works, the inspector will take a second bulb from the box, without replacing the first bulb. If the second bulb does not work, the box will not be shipped. If the second bulb works, the box will be shipped. What is the probability that this box of lightbulbs will not be shipped? (A tree diagram might help)

$(a)\ \frac{42}{90}$ $\quad$ $\underline{(b)}\ \frac{48}{90}$ $\quad$ $(c)\ \frac{70}{100}$ $\quad$ $(d)\ \frac{42}{100}$ $\quad$ $(e)\ \frac{48}{100}$

12 Recall that a poker hand consists of a sample of 5 cards drawn from a deck of 52 cards, with 13 denominations, (Ace, King, Queen, ..., Threes, Twos), and 4 suits, (Diamonds, Hearts, Spades and Clubs). If a poker hand is dealt from a well-shuffled deck, what is the probability that it is a house (three cards from one denomination and two cards from another denomination)?

$\underline{(a)}\quad 13 \cdot C(4,3) \cdot 12 \cdot C(4,2)$ $\quad$ $(b)\quad C(13,3) \cdot C(12,2)$ $\quad$ $(c)\ C(4,3) \cdot C(4,2)$

$(d)\quad 13 \cdot 3 \cdot 12 \cdot 2$ $\quad$ $(e)\quad C(52,3) \cdot C(49,2)$

13 The number of customers waiting in line at the express checkout of Savemore supermarket was counted at the beginning of each 3-min interval between 9 a. m. and noon on Saturday. The data is as follows:

# customers	0	1	2	3	4	5	6	7	8	9	10
Frequency	1	4	2	7	14	8	10	6	3	4	1

What is the realtive frequency of the otcome "5 customers"?

(a) $\frac{2}{60}$ (b) $\frac{8}{55}$ (c) $\frac{5}{60}$ (d) $\frac{5}{55}$ $\underline{(e)}$ $\frac{8}{60}$

14 What is the average number of customers waiting in line, for the sample given in question 13?

(a) 5 $\underline{(b)}$ 4.95 (c) 4 (d) 3.92 (e) 5.1

15 The lives of the lightbulbs produced by the Enligtenment lightbulb company are Normally distributed with mean $\mu = 750$ hours and standard deviation $\sigma = 75$ hours. What is the probability that an Enlightenment bulb selected at random will burn for more than 900 hors?

(a) .6915 (b) .9772 $\underline{(c)}$ .0228 (d) .3085 (e) .5

16 Compute the mean and variance of the Random variable X, where the distribution of X is given by:

k	$\Pr(X = k)$
1	.4
2	.3
3	.2
4	.1

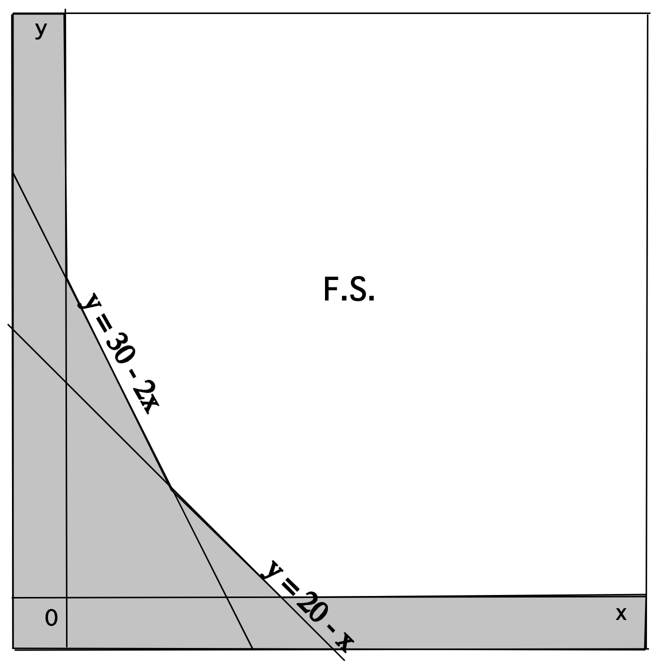
(a) $\mu = 0, \ \sigma^2 = \mathrm{Var}(X) = .2$ (b) $\mu = 2, \ \sigma^2 = \mathrm{Var}(X) = 1.6$ (c) $\mu = 2, \ \sigma^2 = \mathrm{Var}(X) = .2$

(d) $\mu = 0, \quad \sigma^2 = \mathrm{Var}(X) = 1$ $\underline{(e)}$ $\mu = 2, \quad \sigma^2 = \mathrm{Var}(X) = 1$

17 Let Z be a standard normal random variable. What is $Pr(Z \geq -2.5)$?

(a) .9938 (b) .0062 (c) .9842 (d) .0002 (e) .5

18 Find the minimum of the objective function, $4x + 7y$ on the feasible set given below:



(a) 110 (b) 210 (c) 80 (d) 70 (e) 250

19 A Manufacturer wishes to produce two types of souvenirs for a Seaquarium, dolphins and whales (plastic ones of course). Each dolphin will result in a profit of \$1 and each whale will result in a profit of \$1.20. To Manufacture a dolphin requires 2 minutes on machine I and 1 minute on machine II. A whale requires 1 minute on machine I and 3 minutes on machine II. There are 3 hours available on machine I and 5 hours available on machine II for processing the order. Let x be the number of dolphins to be made and let y be the number of whales to be made, which of the following sets of inequalities describe the constraints on production and the profit function?

(a) $$\begin{array}{c} x + 2y \leq 180 \\ 3x + y \leq 300 \\ x \geq 0, \quad y \geq 0 \\ \text{Profit} : x + 1.2y \end{array}$$

(b) $$\begin{array}{c} 2x + y \geq 180 \\ x + 3y \geq 300 \\ x \geq 0, \quad y \geq 0 \\ \text{Profit} : x + 1.2y \end{array}$$

$\underline{(c)}$ $$\begin{array}{c} 2x + y \leq 180 \\ x + 3y \leq 300 \\ x \geq 0, \quad y \geq 0 \\ \text{Profit} : x + 1.2y \end{array}$$

(d) $$\begin{array}{c} x + 2y \geq 180 \\ 3x + y \geq 300 \\ x \geq 0, \quad y \geq 0 \\ \text{Profit} : 1.2x + y \end{array}$$

(e) $$\begin{array}{c} 2x + y \leq 180 \\ x + 3y \leq 300 \\ x \geq 0, \quad y \geq 0 \\ \text{Profit} : 1.2x + y \end{array}$$

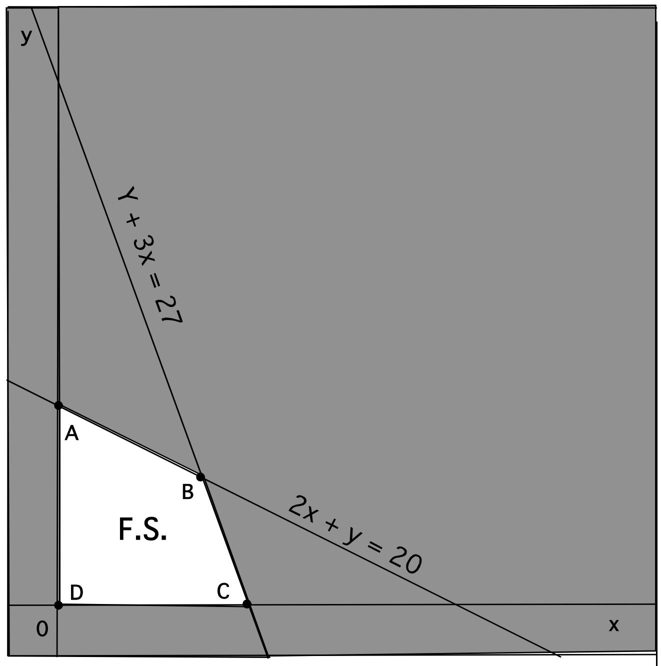
20 What is the product $A \cdot B$ of the following matrices:

$$A = \begin{pmatrix} 3 & 0 \\ 2 & 1 \\ 4 & 2 \end{pmatrix} \qquad B = \begin{pmatrix} -1 & 2 \\ 0 & -1 \end{pmatrix}$$

$\underline{(a)}$ $\begin{pmatrix} -3 & 6 \\ -2 & 3 \\ -4 & 6 \end{pmatrix}$ (b) $\begin{pmatrix} -3 & 6 \\ 2 & 3 \end{pmatrix}$ (c) $\begin{pmatrix} 2 & 2 \\ 2 & 0 \\ 4 & 2 \end{pmatrix}$

(d) $\begin{pmatrix} 1 & 2 \\ -2 & -1 \\ 4 & 2 \end{pmatrix}$ (e) $\begin{pmatrix} 1 & 2 \\ -2 & -1 \end{pmatrix}$

21 What are the vertices of the feasible set shown below?



(a) $\begin{array}{c} A: x=0,\ y=20 \\ B: x=5,\ y=10 \\ C: x=10,\ y=0 \\ D: x=0,\ y=0 \end{array}$

(b) $\begin{array}{c} A: x=0,\ y=27 \\ B: x=7,\ y=6 \\ C: x=10,\ y=0 \\ D: x=0,\ y=0 \end{array}$

(c) $\begin{array}{c} A: x=0,\ y=20 \\ B: x=7,\ y=6 \\ C: x=9,\ y=0 \\ D: x=0,\ y=0 \end{array}$

(d) $\begin{array}{c} A: x=0,\ y=27 \\ B: x=5,\ y=12 \\ C: x=9,\ y=0 \\ D: x=0,\ y=0 \end{array}$

(e) $\begin{array}{c} A: x=0,\ y=27 \\ B: x=5,\ y=10 \\ C: x=10,\ y=0 \\ D: x=0,\ y=0 \end{array}$

22 Consider the points

$$\begin{array}{ll} D: x=5, & y=15 \\ E: x=2, & y=1\ . \\ F: x=8, & y=5 \end{array}$$

Using the feasible set given in Question 21, Which of the following statements are true?

(a) F is in the feasible set

(b) E and F are in the feasible set

(c) D and E are in the feasible set

(d) E is in the feasible set

(e) D and F are in the feasible set

23 Let

$$A = \begin{pmatrix} 3 & 1 \\ 0 & 7 \end{pmatrix}.$$

Which of the following gives the inverse matrix A^{-1}?

(a) $\begin{pmatrix} \frac{7}{20} & -\frac{1}{20} \\ 0 & \frac{3}{20} \end{pmatrix}$ (b) $\begin{pmatrix} \frac{1}{3} & 1 \\ 0 & \frac{1}{7} \end{pmatrix}$ $\underline{(c)}$ $\begin{pmatrix} \frac{7}{21} & -\frac{1}{21} \\ 0 & \frac{3}{21} \end{pmatrix}$

(d) $\begin{pmatrix} \frac{3}{21} & \frac{1}{21} \\ 0 & \frac{7}{21} \end{pmatrix}$ (e) $\begin{pmatrix} \frac{3}{20} & \frac{1}{20} \\ 0 & \frac{7}{20} \end{pmatrix}$

24 Cathy (C) and Ronnie (R) play a game, where both players flip a coin and then reveal the results simultaneously. If both coins show heads, then Cathy pays Ronnie \$2. If both coins show tails, the Ronnie pays Cathy \$3. If Ronnie's coin shows heads and Cathy's coin shows tails, then Ronnie pays Cathy \$1. If Cathy's coin shows heads and Ronnie's coin shows tails, then Cathy pays Ronnie \$3. Which of the folowing matrices, gives the pay-off matrix for Ronnie for this game?

(a)

		C Heads	C Tails
	Heads	2	1
R	Tails	3	3

$\underline{(b)}$

		C Heads	C Tails
	Heads	2	−1
R	Tails	3	−3

(c)

		C Heads	C Tails
	Heads	−2	1
R	Tails	−3	3

(d)

		C Heads	C Tails
	Heads	3	−3
R	Tails	2	−1

(e)

		C Heads	C Tails
	Heads	3	3
R	Tails	2	1

25 Roger Rabbit (R) and Chicken Licken (C) play a zero-sum game where the pay-off matrix for R is given by:

	$C1$	$C2$	$C3$
$R1$	2	1	4
$R2$	5	-1	3
$R3$	1	2	-5

Which of the following statements about this game is true?

(a) R's optimal fixed strategy is to play $R2$ on each play

(b) C's optimal fixed strategy is to play $C3$ on each play

(c) The matrix has a saddle point at $R1C2$

(d) The matrix has no saddle point

(e) R's optimal fixed strategy is to play $R3$ on each play

26 Rose (R) and Colm (C) play a zero-sum game where the pay-off matrix for Rose is given by

	$C1$	$C2$	$C3$
$R1$	2	0	1
$R2$	0	-1	2
$R3$	1	2	-1

If Rose plays the mixed strategy $\begin{pmatrix} .2 & .1 & .7 \end{pmatrix}$, and Colm plays the mixed strategy $\begin{pmatrix} .4 \\ .2 \\ .4 \end{pmatrix}$, what is the expected pay-off for Rose?

(a) .58 *(b)* .44 *(c)* .71 *(d)* .83 *(e)* 1.1

27 Rosita (R) and Carlos (C) play a zero-sum game with pay-off matrix for Rosita given by:

	$C1$	$C2$
$R1$	1	3
$R2$	5	2

If Rosita wants to find her optimal mixed strategy, given that Carlos always plays the best counter-strategy, which of the following linear programming problems must she solve?

(a) Minimize $x+y$ subject to: $x \geq 0, \quad y \geq 0$; $x+5y \geq 1$; $3x+2y \geq 1$

(b) Maximize $x+y$ subject to: $x \geq 0, \quad y \geq 0$; $x+5y \geq 1$; $3x+2y \geq 1$

(c) Minimize $x+y$ subject to: $x \geq 0, \quad y \geq 0$; $x+5y \leq 1$; $3x+2y \leq 1$

(d) Maximize $x+y$ subject to: $x \geq 0, \quad y \geq 0$; $x+3y \geq 1$; $5x+2y \geq 1$

(e) Minimize $x+y$ subject to: $x \geq 0, \quad y \geq 0$; $x+3y \geq 1$; $5x+2y \geq 1$

28 Rosita (R) and Carlos (C) play the zero-sum game given in question 27. If Carlos wants to find his optimal mixed strategy, given that Rosita always plays the best counterstrategy, which of the following linear programming problems must he solve?

(a) Minimize $z+w$ subject to: $z \geq 0, \quad w \geq 0$; $z+3w \leq 1$; $5z+2w \leq 1$

(b) Minimize $z+w$ subject to: $z \geq 0, \quad w \geq 0$; $z+5w \leq 1$; $3z+2w \leq 1$

(c) Maximize $z+w$ subject to: $z \geq 0, \quad w \geq 0$; $z+3w \leq 1$; $5z+2w \leq 1$

(d) Maximize $z+w$ subject to: $z \geq 0, \quad w \geq 0$; $z+3w \geq 1$; $5z+2w \geq 1$

(e) Minimize $z+w$ subject to: $z \geq 0, \quad w \geq 0$; $z+5w \geq 1$; $3z+2w \geq 1$

29 Rosita (R) and Carlos (C) play the zero-sum game given in question 27. Carlos found that the soluution to his linear programming problem from question 28 was

$$z = \frac{1}{13}, \quad w = \frac{4}{13}$$

What is Carlos' optimal mixed strategy given that Rosita plays her best counterstrategy?

(a) $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ (b) $\begin{pmatrix} \frac{4}{5} \\ \frac{1}{5} \end{pmatrix}$ (c) $\begin{pmatrix} \frac{1}{13} \\ \frac{4}{13} \end{pmatrix}$ (d) $\begin{pmatrix} \frac{4}{13} \\ \frac{1}{13} \end{pmatrix}$ $\underline{(e)}$ $\begin{pmatrix} \frac{1}{5} \\ \frac{4}{5} \end{pmatrix}$

30 Rusty (R) and Crusty (C) play a zero-sum game with pay-off matrix for Rusty given by:

	$C1$	$C2$
$R1$	2	1
$R2$	5	-1

If Crusty plays C2 on every play, which of the following mixed strategies gives the highest expected pay-off for Rusty?

(a) $(.2 \;\; .8)$ $\underline{(b)}$ $(1 \;\; 0)$ (c) $(0 \;\; 1)$ (d) $(.5 \;\; .5)$ (e) $(.7 \;\; .3)$