Learning Goals

1. How to add and multiply matrices.
2. Converting a system of linear equations to a matrix equation.
3. Solving two equations in two unknowns with matrices without computer.
4. Solving n equations in n unknowns with unique solution with `R`

Matrices

A **matrix** is a rectangular array of numbers. For example, the following rectangular arrays of numbers are matrices:

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix} \quad B = \begin{bmatrix} 2 & 4 & 7 \\ 5 & 8 & 10 \end{bmatrix} \quad C = \begin{bmatrix} 2 \\ 4 \\ 6 \\ 8 \\ 10 \end{bmatrix} \quad D = \begin{bmatrix} 1 & 3 & 5 & 7 & 9 \end{bmatrix} \quad E = \begin{bmatrix} 4 \\ 1 \\ 47653 \end{bmatrix}$$

Matrices vary in **size**. An $m \times n$ matrix has m rows and n columns. The matrices above have sizes

$$2 \times 2, \quad 2 \times 3, \quad 5 \times 1, \quad 1 \times 5, \quad 3 \times 1$$

respectively.

The numbers in the matrix are called the **entries** of the matrix. Because we may have the same number in more than one position, when we refer to an entry we refer to its position. The (i, j) entry is the entry in the ith row and jth column or the symbol A_{ij} denotes the entry in the i th row and j th column of the matrix A.

Example Using the matrices shown above:

$$A_{12} = 2, \quad A_{21} = 3, \quad C_{31} = 6, \quad B_{23} = 10, \quad E_{31} = 47653.$$

Example Using the matrices defined above find:

$$A_{22} =$$

$$B_{12} =$$

$$D_{13} =$$

$$E_{21} =$$

There are a number of ways to enter a matrix in R. We can use the `matrix(c( , , ...),nrows<-m,ncols<-n)` command for an $m \times n$ matrix, where the vector $c(, , ...)$ is a list of the entries of the matrix by column; the (1,1) entry followed by the (2,1) entry etc.... We could also list the entries by row and use the command `matrix(c( , , ...),nrows<-m,ncols<-n,byrow=TRUE)`.

Example We define the matrices given above in R using lowercase names to avoid conflict:

```
> a<-matrix(c(1,3,2,6),nrows<-2,ncols<-2)
> b<-matrix(c(2,4,7,5,8,10),nrows<-2,ncols<-3,byrow=TRUE)
> c<-matrix(c(2,4,6,8,10),nrows<-5,ncols<-1)
> d<-matrix(c(1,3,5,7,9),nrows<-1,ncols<-5)
> e<-matrix(c(4,1,41653),nrows<-3,ncols<-1)
```

```
> a

     [,1] [,2]
[1,]    1    2
[2,]    3    6
```

```
> b

     [,1] [,2] [,3]
[1,]    2    4    7
[2,]    5    8   10
```

```
> c

     [,1]
[1,]    2
[2,]    4
[3,]    6
[4,]    8
[5,]   10
```

```
> d

     [,1] [,2] [,3] [,4] [,5]
[1,]    1    3    5    7    9
```

```
> e

      [,1]
[1,]     4
[2,]     1
[3,] 41653
```

The (i,j) entry of a matrix `m` in `R` which is `m[i,j]`.

```
> a[2,2]
[1] 6
> b[1,2]
[1] 4
> c[3,1]
[1] 6
> d[1,3]
[1] 5
> e[2,1]
[1] 1
```

Matrices arise naturally in many areas of mathematics. We will use matrix notation to write systems of linear equations as a single matrix equation and we will also use matrix multiplication and inverse matrices to solve systems of equations when appropriate.

Algebra of Matrices

Matrices have arithmetic properties, just like ordinary numbers. We can define an addition and a multiplication for matrices. In both of these binary operations there will be compatibility restrictions on the sizes of the matrices involved.

Adding Two matrices

Before we can add two matrices, they must have the same size. Any two matrices of the same size can be added. We add matrices by adding the corresponding entries. For example:

$$\begin{bmatrix} 2 & 1 & 0 \\ 4 & 0 & 1 \\ 1 & 2 & 3 \\ 0 & 3 & 10 \end{bmatrix} + \begin{bmatrix} 0 & 5 & 7 \\ 4 & 3 & 1 \\ 2 & 2 & 1 \\ 9 & 4 & 0 \end{bmatrix} = \begin{bmatrix} 2+0 & 1+5 & 0+7 \\ 4+4 & 0+3 & 1+1 \\ 1+2 & 2+2 & 3+1 \\ 0+9 & 3+4 & 10+0 \end{bmatrix} = \begin{bmatrix} 2 & 6 & 7 \\ 8 & 3 & 2 \\ 3 & 4 & 4 \\ 9 & 7 & 10 \end{bmatrix}$$

So, if we add two matrices, A and B, the (i, j) entry of $A+B$ is equal to $A_{ij}+B_{ij}$.

To add matrices in `R` we simply use the `+` symbol. For example:

```
> x<-matrix(c(2,4,1,0,1,0,2,3,0,1,3,10),nrows<-4,ncols<-3)
> y<-matrix(c(0,4,2,9,5,3,2,4,7,1,1,0),nrows<-4,ncols<-3)
> x+y
     [,1] [,2] [,3]
[1,]    2    6    7
[2,]    8    3    2
[3,]    3    4    4
[4,]    9    7   10
```

Example Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$ $\qquad D = \begin{bmatrix} 1 & 0 \\ 5 & 9 \end{bmatrix}$

Then $\qquad A + D =$

Matrix Multiplication (A mild form)

We start by multiplying a row matrix by a column matrix. Here the number of entries in the row must equal the number of entries in the column. The general formula is given by

$$\begin{bmatrix} a_1 & a_2 & \cdots & a_{n-1} & a_n \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_{n-1} \\ b_n \end{bmatrix} = a_1 \cdot b_1 + a_2 \cdot b_2 \ \cdots \ a_{n-1} \cdot b_{n-1} + a_n \cdot b_n$$

Example Calculate the following:

$$\begin{bmatrix} 1 & 2 & 2 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ 1 \\ 5 \\ 3 \end{bmatrix} = \qquad\qquad \begin{bmatrix} 1 & 3 & 5 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix} =$$

$$\begin{bmatrix} 3 & 8 & 7 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 1 \\ 5 \end{bmatrix} = \qquad\qquad \begin{bmatrix} 1 & 8 & 4 & 5 & 0 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 4 \\ 5 \\ 1 \\ 1 \end{bmatrix} =$$

Note that when you multiply a row by a column, you just get a number or a 1×1 matrix. Above we have (in order),

A 1×5 matrix multiplied by a 5×1 matrix gives a 1×1 matrix.
A 1×3 matrix multiplied by a 3×1 matrix gives a 1×1 matrix.
A 1×4 matrix multiplied by a 4×1 matrix gives a 1×1 matrix.
A 1×6 matrix multiplied by a 6×1 matrix gives a 1×1 matrix.

Compatibility In order to multiply two matrices, we must have compatible sizes. Let A be an $m \times p$ matrix and let B be a $q \times n$ matrix. Then I can form the product AB only if $p = q$. If $p = q$, then AB will be an $m \times n$ matrix.

Example: let:

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix} \quad B = \begin{bmatrix} 2 & 4 & 7 \\ 5 & 8 & 10 \end{bmatrix} \quad C = \begin{bmatrix} 2 \\ 4 \\ 6 \\ 8 \\ 10 \end{bmatrix} \quad D = \begin{bmatrix} 1 & 3 & 5 & 7 & 9 \end{bmatrix} \quad E = \begin{bmatrix} 4 \\ 1 \\ 47653 \end{bmatrix}$$

Which of the following matrix products can be formed and if it can be formed, what size is the matrix?

Product	AB	BC	AC	DC	AE	EA	EB	BE
Possible Y/N								
Size								

Rather than study general matrix multiplication, we will limit our study of matrix multiplication to that which will occur in solving systems of linear equations. Our goal is to be able to calculate products of the form:

$$AB \qquad \text{and} \qquad BC,$$

where A is a $1 \times m$ row matrix, B is an $m \times n$ matrix and C is a column matrix of the form $n \times 1$.

To multiply the $1 \times m$ row matrix A by the $m \times n$ matrix B, we multiply the row matrix A by the columns of B to get the entries of AB. Specifically, AB is a $1 \times n$ matrix (a row matrix) the $(1, j)$ entry of AB is the row matrix A multiplied by the j th column of B.

Example Let $A = \begin{bmatrix} 1 & 2 \end{bmatrix}$ $\quad B = \begin{bmatrix} 2 & 4 & 7 \\ 5 & 8 & 10 \end{bmatrix}$

Since A is a 1×2 matrix and B is a 2×3 matrix, AB will be a 1×3 matrix.

To calculate the $(1,1)$ entry of AB, we multiply the row matrix A by column 1 of B

$$\begin{array}{c} A \\ \begin{bmatrix} 1 & 2 \end{bmatrix} \end{array} \begin{array}{c} B \\ \begin{bmatrix} 2 & 4 & 7 \\ 5 & 8 & 10 \end{bmatrix} \end{array} = \begin{bmatrix} 1\cdot 2 + 2 \cdot 5 & - & - \end{bmatrix} = \begin{array}{c} AB \\ \begin{bmatrix} 12 & - & - \end{bmatrix} \end{array}$$

To calculate the $(1,2)$ entry of AB, we multiply the row matrix A by Column 2 of B.

$$\begin{array}{c} A \\ \begin{bmatrix} 1 & 2 \end{bmatrix} \end{array} \begin{array}{c} B \\ \begin{bmatrix} 2 & 4 & 7 \\ 5 & 8 & 10 \end{bmatrix} \end{array} = \begin{bmatrix} 12 & 1\cdot 4 + 2 \cdot 8 & - \end{bmatrix} = \begin{array}{c} AB \\ \begin{bmatrix} 12 & 20 & - \end{bmatrix} \end{array}$$

To calculate the $(1,3)$ entry of AB, we multiply Row 1 of A by Column 3 of B.

$$\begin{array}{c} A \\ \begin{bmatrix} 1 & 2 \end{bmatrix} \end{array} \begin{array}{c} B \\ \begin{bmatrix} 2 & 4 & 7 \\ 5 & 8 & 10 \end{bmatrix} \end{array} = \begin{bmatrix} 12 & 20 & 1\cdot 7 + 2 \cdot 10 \end{bmatrix} = \begin{array}{c} AB \\ \begin{bmatrix} 12 & 20 & 27 \end{bmatrix} \end{array}$$

Example Let

$$A = \begin{bmatrix} 1 & 2 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} 3 & 1 \\ 0 & 2 \\ 4 & 1 \end{bmatrix}$$

Calculate AB

To multiply the $m \times n$ matrix B by the $n \times 1$ column matrix C, we multiply each row of B by the column matrix C to get the rows of BC. In particular, BC is a $m \times 1$ column matrix where the $(k,1)$ entry of BC is the kth row of A multiplied by the column matrix B.

Example Let $B = \begin{bmatrix} 2 & 4 & 7 \\ 5 & 8 & 10 \end{bmatrix}$ and $C = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$

To calculate the $(1,1)$ entry of BC, we multiply row 1 of B by the column matrix C.

$$\begin{array}{c} B \\ \begin{bmatrix} 2 & 4 & 7 \\ 5 & 8 & 10 \end{bmatrix} \end{array} \begin{array}{c} C \\ \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \end{array} = \begin{bmatrix} 2\cdot 1 + 4 \cdot 0 + 7 \cdot (-1) \\ - \end{bmatrix} = \begin{array}{c} BC \\ \begin{bmatrix} -5 \\ - \end{bmatrix} \end{array}$$

To calculate the $(2,1)$ entry of BC, we multiply row 2 of B by the column matrix C.

$$\overset{B}{\begin{bmatrix} {\color{red}2} & {\color{red}4} & {\color{red}7} \\ {\color{blue}5} & {\color{blue}8} & {\color{blue}10} \end{bmatrix}} \overset{C}{\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}} = \begin{bmatrix} {\color{red}-5} \\ {\color{blue}5} \cdot 1 + {\color{blue}8} \cdot 0 + {\color{blue}10} \cdot (-1) \end{bmatrix} = \overset{BC}{\begin{bmatrix} {\color{red}-5} \\ {\color{blue}-5} \end{bmatrix}}$$

To multiply two compatible matrices `b` and `c` in `R` with matrix multiplication, we use `a`. For example:

```
> a<-matrix(c(1,2),nrows<-1,ncols<-2)
> b<-matrix(c(2,5,4,8,7,10),nrwos<-2,ncols<-3)
> c<-matrix(c(1,0,-1),nrows<-3,ncols<-1)
> a%*%b

     [,1] [,2] [,3]
[1,]   12   20   27

> b%*%c

     [,1]
[1,]   -5
[2,]   -5
```

Example Let $B = \begin{bmatrix} 3 & 1 \\ 0 & 2 \\ 4 & 1 \end{bmatrix}$ and $C = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$.

Calculate BC.

Converting a system of Linear Equations to a Matrix Equation

Given a system of linear equations in two unknowns

$$\begin{cases} -2x + 4y &= 2 \\ -3x + 7y &= 7 \end{cases}$$

we can write it in matrix form as a single equation $AX = B$, where

$$A = \begin{pmatrix} -2 & 4 \\ -3 & 7 \end{pmatrix}, \quad X = \begin{pmatrix} x \\ y \end{pmatrix}, \quad B = \begin{pmatrix} 2 \\ 7 \end{pmatrix}.$$

When we multiply we get

$$AX = \begin{pmatrix} -2x + 4y \\ -3x + 7y \end{pmatrix}$$

a 2×1 matrix. When we set this equal to the matrix $B = \begin{pmatrix} 2 \\ 7 \end{pmatrix}$, we get our two equations back again by equating the elements of each matrix, thus getting our linear system back again:

$$\begin{cases} -2x + 4y &= 2 \\ -3x + 7y &= 7 \end{cases}$$

Example Convert the system of linear equations shown below to a matrix equation of the form $AX = B$.

$$\begin{cases} 2x_1 + 4x_2 &= 2 \\ 3x_1 + 0x_2 &= 2 \end{cases}$$

The same approach can be used for systems of equations with any number of variables. In this case, the system of n equations

$$\begin{cases} a_{1,1}x_1 + a_{1,2}x_2 + \cdots + a_{1,n}x_n &= b_1 \\ a_{2,1}x_1 + a_{2,2}x_2 + \cdots + a_{2,n}x_n &= b_2 \\ \vdots & \vdots \\ a_{m,1}x_1 + a_{m,2}x_2 + \cdots + a_{m,n}x_n &= b_m \end{cases}$$

can be written as a matrix equation $AX = B$ where

$$A = \begin{pmatrix} a_{1,1} & a_{1,2} & \dots & a_{1,n} \\ a_{2,1} & a_{2,2} & \dots & a_{2,n} \\ \vdots & \vdots & & \vdots \\ a_{n,1} & a_{n,2} & \dots & a_{n,n} \end{pmatrix}, \quad X = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}, \quad B = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix}$$

Example Convert the system of linear equations shown below to a matrix equation of the form $AX = B$.

$$\begin{cases} 2x_1 + 4x_2 + 3x_3 + x_4 &= 2 \\ x_1 + x_2 + 0x_3 + 2x_4 &= 4 \\ 0x_1 + x_2 + x_3 + 0x_4 &= 5 \\ 3x_1 + 0x_2 + x_3 + 2x_4 &= 2 \end{cases}$$

Multiplication and multiplicative inverses for square matrices

If a matrix has the same number of rows as it does columns, it is called a square matrix. We will see below that the set/system of all $n \times n$ matrices has all of the characteristics of the real number system that are necessary to perform basic algebra to solve for unknown variables. We will use this algebra to solve matrix equations resulting from a system of equations.

Example The 2×2, 3×3 and 4×4 matrices shown below are square matrices.

$$\begin{bmatrix} 2 & 1 \\ 4 & 0 \end{bmatrix} \qquad \begin{bmatrix} 2 & 1 & 0 \\ 4 & 0 & 1 \\ 1 & 2 & 3 \end{bmatrix} \qquad \begin{bmatrix} 2 & 1 & 0 & 5 \\ 4 & 0 & 1 & 2 \\ 1 & 2 & 3 & 9 \\ 0 & 3 & 10 & 0 \end{bmatrix}$$

Note that for any given n, we can add, subtract and multiply two $n\times n$ matrices to get another $n \times n$ matrix. The zero $n \times n$ matrix (an $n \times n$ matrix where all entries are 0) which we denote by $0_{n\times n}$ has the properties that

$$A + 0_{n\times n} = A - 0_{n\times n} = A$$

and

$$A - A = 0_{n\times n}$$

for any $n \times n$ matrix A. Thus $0_{n\times n}$ plays the same role in the system of $n \times n$ matrices as that played by the number 0 in the real number system.

Algebra with matrices This allows us for example to solve for X (a 2×2) matrix in the following equation as we would in regular algebra:

$$\begin{bmatrix} 2 & 1 \\ 4 & 0 \end{bmatrix} + X = \begin{bmatrix} 0 & 1 \\ 3 & 2 \end{bmatrix}$$

We subtract the matrix $\begin{bmatrix} 2 & 1 \\ 4 & 0 \end{bmatrix}$ from both sides to get

$$\begin{bmatrix} 2 & 1 \\ 4 & 0 \end{bmatrix} + X - \begin{bmatrix} 2 & 1 \\ 4 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 3 & 2 \end{bmatrix} - \begin{bmatrix} 2 & 1 \\ 4 & 0 \end{bmatrix}$$

this gives

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} + X = \begin{bmatrix} -2 & 0 \\ -1 & 2 \end{bmatrix} \quad \text{and thus} \quad X = \begin{bmatrix} -2 & 0 \\ -1 & 2 \end{bmatrix}$$

Indentity Matrix In the real number system, the number 1 is to multiplication as the number 0 is to addition. We also have an analogue of the number 1 in an $n \times n$ matrix system which we denote by $I_{n\times n}$. It is the matrix with 1's on the main diagonal and 0's elsewhere. We show $I_{2\times2}$, $I_{3\times3}$ and $I_{4\times4}$ below.

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

For an $n \times n$ matrix A, we have

$$A \cdot I_{n\times n} = A = I_{n\times n} \cdot A.$$

Inverse Matrices and Determinants In the real number system, division is to multiplication what subtraction is to addition. However, we have some restrictions on division in that we can never divide by the number 0. We also have division in an $n \times n$ matrix system with some restrictions. For each $n \times n$ matrix, there is a number associated to the matrix called the determinant. So for an $n \times n$ matrix A, we denote the **determinant by det$A$.** <u>We can only divide by matrices for which det$A \neq 0$.</u> We will show how to calculate this determinant for a 2×2 matrix below. The formula is more complicated for $n \times n$ matrices with $n \geq 3$ and we will use `R` to calculate determinants when necessary.

Determinant and Inverse of a 2×2 matrix For a two by two matrix, $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, we have $\det A = ad - bc$. If this number is non-zero, we can create a matrix which "cancels" the matrix A when multiplied by A, namely the matrix

$$\left(\frac{I}{A} = \right) \quad A^{-1} = \frac{1}{\det A} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \frac{1}{\text{ad - bc}} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

This inverse matrix has the following properties:

$$A \cdot A^{-1} = I_{2\times2} = A^{-1} \cdot A.$$

To find the determinant of a square matrix `a` using `R`, we use the command `det(a)` and to find the inverse of a square matrix using `R` we use the command `solve(a)`. For example:

```
> a<-matrix(c(2,4,1,0),nrows<-2,ncols<-2)
> b<-matrix(c(2,4,1,1,0,2,0,1,3),nrows<-3,ncols<-3)
> a

     [,1] [,2]
[1,]    2    1
[2,]    4    0
```

```
> b

     [,1] [,2] [,3]
[1,]    2    1    0
[2,]    4    0    1
[3,]    1    2    3

> det(a)

[1] -4

> det(b)

[1] -15

> solve(a)

     [,1]  [,2]
[1,]    0  0.25
[2,]    1 -0.50

> solve(b)

           [,1] [,2]        [,3]
[1,]  0.1333333  0.2 -0.06666667
[2,]  0.7333333 -0.4  0.13333333
[3,] -0.5333333  0.2  0.26666667
```

Example Let $A = \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}$. Find det A, determine if A has an inverse and if so find it.

Using matrix inverses and `R` to solve systems of equations

(See 2.4, Goldstein, Schneider and Siegel

Similarly, if A is an $n \times n$ matrix with det$A \neq 0$, we can create an inverse matrix A^{-1} with the properties

$$A \cdot A^{-1} = I_{n \times n} = A^{-1} \cdot A.$$

Canceling matrices in Matrix Equations When solving for the unknown x in the equation $2x = 4$ where x is a real number, we divide across by 2 (multiply by $\frac{1}{2}$) to cancel the 2; we get

$$\frac{1}{2} \cdot 2x = \frac{1}{2} \cdot 4 \quad \text{which gives} \quad x = 2.$$

Similarly, if we have a matrix equation, $AX = B$, where A is an $n \times n$ matrix, X and B are $n \times m$ matrices and set $A \neq 0$, we can solve for the unknown matrix X in the matrix equation

$$AX = B$$

by multiplyingg both sides by A^{-1} to get

$A^{-1}AX = A^{-1}B$ which gives $I_{n\times n}X = A^{-1}B$ which in turn gives $X = A^{-1}B$.

Example Solve the system of equations shown below by solving the corresponding matrix equation.

$$\begin{array}{ccccc} 2x_1 & + & x_2 & = & 3 \\ x_1 & + & x_2 & = & 5 \end{array}$$

If a system of linear equations has matrix form $AX = B$, where A is an invertible $n \times n$ matrix, then there is a unique solution to the system of equations. That solution is given by $X = A^{-1}B$. To solve a matrix equation of the form $AX = B$ where A is an invertible $n \times n$ matrix with $n > 2$, we will use `R`.

Solving a system of equations with a unique solution with `R` using `solve(a,b)`

Suppose we wanted to solve for X in the matrix equation

$$\begin{bmatrix} 2 & 3 & 4 \\ 1 & 1 & 1 \\ 1 & 5 & 10 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 9 \\ 3 \\ 16 \end{bmatrix},$$

We enter the matrices $m = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 1 & 1 \\ 1 & 5 & 10 \end{bmatrix}$ and $b = \begin{bmatrix} 9 \\ 3 \\ 16 \end{bmatrix}$ in `R` and use the command `solve(m,b)` to solve for $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

```
> m<-matrix(c(2,1,1,3,1,5,4,1,10), nrows<-3,ncols<-3)
> b<-matrix(c(9,3,16),nrows<-3,ncols<-1)
> solve(m,b)

     [,1]
[1,]    1
[2,]    1
[3,]    1
```

Example Convert the system of linear equations shown below to a matrix equation of the form $AX = B$ and solve the system using R.

$$\begin{cases} x + 2y + z &= 2 \\ x - y + 2z &= 4 \\ x - y - z &= 5 \end{cases}$$

Example Convert the system of linear equations shown below to a matrix equation of the form $AX = B$ and solve the system using R.

$$\begin{cases} 2x_1 + 4x_2 + 3x_3 + x_4 & = & 2 \\ x_1 + x_2 + 0x_3 + 2x_4 & = & 4 \\ 0x_1 + x_2 + x_3 + 0x_4 & = & 5 \\ 3x_1 + 0x_2 + x_3 + 2x_4 & = & 2 \end{cases}$$

R commands; `a,b,x` are matrices below

1. `matrix(c( , , ...),nrows<-m,ncols<-n)` gives an $m \times n$ matrix, where the vector $c(, , ...)$ is a list of the entries of the matrix by column.
2. `matrix(c( , , ...),nrows<-m,ncols<-n,byrow=TRUE)` gives an $m \times n$ matrix, where the vector $c(, , ...)$ is a list of the entries of the matrix by row.
3. The (i,j) entry of a matrix `m` in `R` which is `m[i,j]`.
4. `a + b` : add `a` and `b`.
5. `a%*%b` : matrix multiplication of `a` and `b`.
6. `det(a)` : determinant of `a`.
7. `solve(a)` :inverse of `a`.
8. `solve(a, b)` :gives solution for x in matrix equation `ax = b`.