Kinematic Equations

- Descriptions of Motion (words → sentences)

\[ x, \text{ velocity } v = \frac{\Delta x}{\Delta t}, \text{ acceleration } a = \frac{\Delta v}{\Delta t} \]

\[ v = v_i + at, \quad a = \frac{\Delta v}{\Delta t}, \quad a \Delta t = \Delta v = v_f - v_i \]
\[ v = \frac{\Delta x}{\Delta t}, \quad v \Delta t = \Delta x \]

\[ \Delta x = x_f - x_i \]

\[ \text{total Area} = \text{total distance} \]
\[ A_1 = \frac{1}{2} (t_f - t_i) (v_f - v_i) \]

\[ a = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i} \]

\[ a (t_f - t_i) = (v_f - v_i) \]

\[ A_2 = v_i (t_f - t_i) \]

\[ A_1 = \frac{1}{2} a (t_f - t_i)^2, \quad t_f = t, \quad t_i = 0 \]

\[ A_1 + A_2 = \frac{1}{2} at^2 + v_i t = x_f - x_i \]

\[ \Rightarrow x = x_i + v_i t + \frac{1}{2} at^2 \]

\[ x = x_i + v_i t + \frac{1}{2} at^2 \]
Kinematic Equations

- Descriptions of Motion (words → sentences)
- Summary:

$$\dot{v} = \dot{v}_i + at$$

$$x = x_i + v_i t + \frac{1}{2} at^2$$
Helpful Hints for Kinematics

• **Time** is the key to kinematics:
  – *the* independent variable
  – horizontal axis for motion graphs
• For problem solving:
  – you can always refer everything back to the time at which it happens
  – simultaneous events occur at the same time
  – multiple objects must be referenced to the *same* coordinate system
A ball is thrown upward with an initial velocity of 20 m/s.

a) how long is the ball in the air?

b) What is the greatest height reached by the ball?

\[ v_{\text{top}} = 0 \quad a = g \quad \text{gravity} \]

\[ \text{v} = v_i + at \quad \text{at top} \quad v = 0 \quad 20 \text{ m/s} - gt_{\frac{1}{2}}, \quad t_{\frac{1}{2}} = \frac{20 \text{ m/s}}{9.8 \text{ m/s}^2} = 2.04 \text{ sec} \]

\[ t_{\text{total}} = 2t_{\frac{1}{2}} = 4.08 \text{ sec} \]

\[ y = y_i + v_i t + \frac{1}{2} at^2 \quad y_{\text{max}} = v_i t_{\frac{1}{2}} - \frac{1}{2} gt_{\frac{1}{2}}^2 = (20)(2.04) - \frac{1}{2}(9.8)(2.04)^2 \]

\[ = 20.4 \text{ m} \]
A top-fuel drag racing car can reach a speed of 100 mph in the first second of a race. (100 mph = 44.7 m/s)

(a) Find the acceleration of the car, assuming that the acceleration is constant

\[ v_f = v_i + at, \quad 44.7 = 0 + a(1) \]

(b) If the car continued at this acceleration, how fast would it be going at the end of the quarter-mile track? (0.25 miles is approximately 0.42 km)

\[ a = \frac{44.7 \text{ m/s}^2}{4.69 \text{ s}} \]

\[ t = \sqrt{\frac{2d}{a}} = 4.33 \text{ sec} \]

\[ V_f = at = (44.7 \text{ m/s})(4.33 \text{ sec}) = 193 \text{ m/s} \]

\[ 430 \text{ mph} \]

\[ V_{rc} = 330 \text{ mph} \]
Motion in 2 and 3 Dimensions

- Update: position, displacement, velocity, acceleration are *vectors* (meaning, they don’t just point in one direction)

- Problems become tractable by looking at the individual *components* of the vector equations
**Position:**

\( (x_0, y_0) \) \rightarrow \( (x_f, y_f) \)

**Displacement:**

\( \Delta x \rightarrow \Delta \vec{r} \)

\[ \Delta x = x_f - x_i \]

\[ \Delta y = y_f - y_i \]
velocity: \[ \vec{v} = \frac{\Delta \vec{r}}{\Delta t} \]

acceleration: \[ \vec{a} = \frac{\Delta \vec{v}}{\Delta t} \]

\[ \Delta V_x = \frac{x_f - x_i}{\Delta t}, \quad \Delta V_y = \frac{y_f - y_i}{\Delta t} \]

\[ a_x = \frac{\Delta V_x}{\Delta t}, \quad a_y = \frac{\Delta V_y}{\Delta t} \]