Lecture 1 : Plurality and Runoff Methods

Plurality Method

One very simple method of Voting is
The Plurality Method With this method, each voter selects one candidate or choice on the ballot. The winner is the candidate or choice with the most votes.

Example 1 A committee of 10 people (with names A, B, ... , J) must vote on a venue for their next Gymnastics competition. The choices are Indianapolis, South Bend, Fort Wayne, Terre Haute. The committee uses the plurality method of voting, and their ballots are given in the following table:

	A	B	C	D	E	F	G	H	I	J
Indianapolis		X				X	X			X
South Bend	X			X				X		
Fort Wayne									X	
Terre Haute			X		X					

(a) Which venue did they choose?

(b) Just before the votes are counted, the committee discovers that Terre Haute is not available. If those who votes for Terre Haute replace their ballots, which venues could be selected?

(a) They chose Indianapolis, with 4 votes = highest number of votes.

(b) There are two votes for Terre Haute.
If Indianapolis got those either one of those votes it would be selected.
If South Bend got both of those votes it would be selected.
Since Fort Wayne has just one vote, it could not be selected even if it got both of the Terre Haute votes.

Sometimes this method leads to a **tie** as in the following table. This is less likely when there are large numbers of voters.

	A	B	C	D	E	F	G	H	I	J
Indianapolis		X				X	X			X
South Bend	X			X				X	X	
Fort Wayne										
Terre Haute			X		X					

Minimum Votes Needed to Win

Consider the following situation. There are five candidates for the club player of the year award. The decision is made by 100 voters using the plurality method. After the first sixty votes have been cast, the votes are as follows:

	# Votes
Morris	2
Peterson	21
O'Neill	5
McNulty	13
Dyer	19

What is the minimal number of the remaining votes Peterson needs in order to be guaranteed to win?

Solution Dyer is in second place behind Peterson. In the worst possible scenario, Dyer would get all of the votes that did not go to Peterson. Dyer needs only 2 votes to catch up with Peterson and if Dyer gets 20(one more than half) of the remaining 38 votes, he will beat Peterson. If Peterson gets 20 of the remaining 40 votes, this scenario cannot happen and Peterson is sure of a win.

Alternative Solution Let x denote the number of votes Peterson needs to ensure at least a tie with Dyer for first place. Then

$$x + 21 = 19 + (40 - x)$$

We solve for x;

$$x + 21 = 59 - x$$

$$2x = 38$$

$$x = 19$$

Therefore if Peterson gets $20 = 19 + 1$ votes from the remaining forty, he is guaranteed to win.

Problems With The Plurality Method

Splitting vote on similar choices

If there are just two choices or candidates and the plurality method is used, then the popular choice is guaranteed to win. However if there are more than two choices then it may happen that more extreme choices may win over many similar, but popular choices.

Example 2 Suppose a group of 10 people, many of whom like camping and hiking activities are deciding on where to spend fall break. The options are Camping and Hiking in Colorado, Camping and Hiking in California, Camping and Hiking in Washington, Camping and Hiking in Ireland, Disneyworld. Using the Plurality method the group may end up with a vote like this

Camping and Hiking in Colorado	2
Camping and Hiking in California	2
Camping and Hiking in Washington	2
Camping and Hiking in Ireland	1
Disneyworld	3

Clearly Camping and Hiking is preferred to Disneyworld, but beacause there are so many similar options for Camping and Hiking, the group ends up going to Disneyworld.

Strategic Voting

Another problem with the plurality voting system is that there may be an incentive for the voters supporting a weak choice to vote strategically.

Strategic voting occurs when a voter votes in a way that does not reflect their true preferences in an attempt to improve the outcome of the election/poll.

In Example 1 above, suppose that the voter who prefers Fort Wayne knows that nobody else will vote for Fort Wayne. Suppose also that this voter prefers South Bend to Indianapolis, how can he/she improve the chances that South Bend will win?

Runoff Voting

Because of the problems with plurality method, a runoff election is often used.

In a **Runoff Election**, a plurality vote is taken first.

1. If one of the candidates has more than 50% of the votes, that candidate wins.

2. If no candidate has has more than 50% of the votes, a second round of plurality voting occurs with a designated number of the top candidates.

3. The process continues until one of the candidates has more than 50% of the votes.

Example Consider the example above, where a group of 10 people, are deciding on where to spend fall break. We decide on the following runoff voting system:

1. If one of the choices has more than 50% of the votes in round 1, that choice wins.

2. If no choice has has more than 50% of the votes in round 1, a second round of plurality voting occurs where the choices are those which received first and second place in round 1.

3. The process continues until one of the candidates has more than 50% of the votes.

Is this new voting process likely to change the outcome?

Example: Olympic Voting

The selection of the site for the Olympic Games is made by the International Olympic Committee. The voting process calls for a plurality election with a runoff between all of the candidates except the one in the last place. (This is known as the Hare Method). A number of controversial results have led to suspicions about strategic voting in the past. The results of the election for the location of the 2016 summer olympics are shown below.

Election of the Host City of the 2016 Summer Olympics — ballot results

Candidate City	Country (NOC)	Round 1	Round 2	Round 3
Rio de Janeiro	Brazil (BRA)	26 (27.66%)	**46** (48.42%)	**66** (67.35%)
Madrid	Spain (ESP)	**28** (29.79%)	29 (30.53%)	32 (32.65%)
Tokyo	Japan (JPN)	22 (23.40%)	20 (21.05%)	–
Chicago	United States (USA)	18 (19.15%)	–	–

121st IOC Session	Vote details	Round 1	Round 2	Round 3
Copenhagen - Denmark	Eligible	95	97	99
	Participants	94	96	98
	Abstentions	0	1	0
	Valid ballots	94	95	98

Members unable to vote

Members from countries with candidate cities	Other members
Anita L. Defrantz · James L. Easton · Chiharu Igaya · Shun-Ichiro Okano · João Havelange · Carlos Arthur Nuzman · Juan Antonio Samaranch Jr.	Jacques Rogge (IOC president) · Lee Kun-hee (suspended) · Alpha Ibrahim Diallo (absent) · Saku Koivu (absent)

Can you find evidence of strategic voting?

The results of the election in other years are attached at the end of the lecture. Check to see if you can find evidence of strategic voting in the election process for other years.

Example There are three candidates for the President of the the Notre Dame Squash Club, Roberts, Williams and Peters. The voting system used is a runoff election with the person with the least votes

eliminated after each round. (In the event of a tie for last place a tie-break vote is cast). The results after round 1 are as follows:

Roberts	22
Williams	18
Peters	10
Total	50

(a) Assuming that those who voted for Roberts and Williams in Round 1 will also vote for Roberts and Williams in Round 2, how many of Peters' supporters need to vote for Roberts in order for Roberts to win the election?

Preference Ranking

In most voting situations, each voter has an order of preference of the candidates. Such an ordering is called a **Preference Ranking**. The voter may have to put some thought into making such a preference ranking and it may change over time.

The voting systems discussed below which use preference rankings make the following assumptions about them:
1. Each voter has a preference ranking that orders all candidates from most preferred to least preferred. (we assume that in the case of indifference or lack of knowledge of the candidates, the voter will choose a ranking randomly).
2. If a voter has ranked one candidate higher than another, then if the voter must choose between the two candidates, the voter would choose the higher ranked one.
3. The order of the preferences is not changed by the elimination of one or more candidates.

Example A fourth grade class is asked to rank their preferences for a field trip to a game of football basketball or baseball. The preference rankings of the voters are presented in a table below showing the number of voters with each preference in the top row.

# voters	**1**	**3**	**3**	**2**	**4**	**5**
football	1	1	3	2	4	3
basketball	2	4	1	4	1	4
baseball	3	2	4	3	2	1
soccer	4	3	2	1	3	2

(a) In a plurality election, which option would win?

(b) In a plurality election with a runoff between the top two finishers, what would the outcome be?

(b) In a plurality election with a runoff between the top two finishers, could the two voters who ranked soccer first achieve a preferable outcome by voting strategically if the other voters voted as indicated in the table?

Instant Runoff Voting (IRV) (used in deciding winner of Oscars)

In an Instant Runoff election,

1. each voter ranks the list of candidates in order of preference. The candidates are ranked in ascending order with a "1" next to the most preferred candidate, a "2" next to the second most preferred candidate and so forth.

(In some implementations, the voter ranks as many or as few choices as they wish while in others they are required to rank all of the candidates or a prescribed number of them.)

2. In the initial count, the first preference of each voter is counted and used to order the candidates. Each first preference counts as one vote for the appropriate candidate.

3. Once all the first preferences are counted, if one candidate holds a majority (more than 50% of votes cast), that candidate wins. Otherwise the candidate who holds the fewest first preferences is eliminated.

(If there is an exact tie for last place in numbers of votes, tie-breaking rules determine which candidate to eliminate.)

4. Ballots assigned to eliminated candidates are recounted and assigned to one of the remaining candidates based on the next preference on each ballot.

5. The process repeats until one candidate achieves a majority (more than 50%) of votes cast for continuing candidates. Ballots that 'exhaust' all their preferences (all its ranked candidates are eliminated) are set aside.

Example In an instant runoff election which of the candidates in the previous example would win?

# voters	**1**	**3**	**3**	**2**	**4**	**5**
football	1	1	3	2	4	3
basketball	2	4	1	4	1	4
baseball	3	2	4	3	2	1
soccer	4	3	2	1	3	2

Election of the host city for the Summer Olympics

Location for 2012 Olympics

Candidate City	Round 1	Round 2	Round 3	Round 4
London	22	27	39	54
Paris	21	32	33	50
Madrid	20	25	31	–
New York	19	16	–	–
Moscow	15	–	–	–
Total	97	100	103	104

Location for 2008 Olympics

Candidate City	Round 1	Round 2	Round 3	Round 4
Beijing	44	56	–	–
Toronto	20	22	–	–
Paris	15	18	–	–
Istanbul	17	9	–	–
Osaka	6	–	–	–
Total	102	105	–	–

Location for 2004 Olympics

Candidate City	Round 1	Round 2	Round 3	Round 4
Athens	32	38	52	66
Rome	23	28	35	41
Stockholm	20	19	–	–
Cape Town	16	22	20	–
Buenos Aires	16	–	–	–
Total	107	107	107	107

(Buenos Aires was eliminated in round one with a tie-break vote vs. Cape Town. The result was 62-44.)

Location for 2000 Olympics

Candidate City	Round 1	Round 2	Round 3	Round 4
Sydney	30	30	37	45
Beijing	32	37	40	43
Manchester	11	13	11	–
Berlin	9	9	–	–
Istanbul	7	–	–	–
Total	89	89	88	88

Election of the host city for the Summer Olympics

Location for 1996 Olympics continued

Candidate City	Round 1	Round 2	Round 3	Round 4	Round 5
Athens	23	23	26	30	35
Atlanta	19	20	26	34	51
Toronto	14	17	18	22	–
Melbourne	12	21	16	–	–
Manchester	11	5	–	–	–
Belgrade	7	–	–	–	–
Total	86	86	86	86	86

Location for 1992 Olympics continued

Candidate City	Round 1	Round 2	Round 3
Barcelona	29	37	47
Paris	19	20	23
Brisbane	11	9	10
Belgrade	13	11	5
Birmingham	8	8	–
Amsterdam	5	–	–
Total	85	85	85

(Barcelona had more than 50% of the votes in the third round and thus the voting was terminated with a win for Barcelona.)