Name: NETTY THE INCREDible

Instructor: ______________________

Math 10170, Exam I
March 1, 2016

- The Honor Code is in effect for this examination. All work is to be your own.
- You may use your Calculator.
- The exam lasts for 50 minutes.
- Be sure that your name is on every page in case pages become detached.
- Be sure that you have all 11 pages of the test.

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1.	(a)	(b)	(c)	(d)	(e)
2.	(a)	(b)	(c)	(d)	(e)
3.	(a)	(b)	(c)	(d)	(e)
4.	(a)	(b)	(c)	(d)	(e)
5.	(a)	(b)	(c)	(d)	(e)
6.	(a)	(b)	(c)	(d)	(e)
7.	(a)	(b)	(c)	(d)	(e)

Please do NOT write in this box.

Multiple Choice	______
8.	______
9.	______
10.	______
11.	______
12.	______
Total	______

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Multiple Choice

1.(6 pts.) Five athletes took part in a pentathlon event at at an informal college athletic meet. Each athlete participated in 5 events, fencing (F), swimming (S), shooting (Sh), equestrian(E), cross country (CC). Instead of the usual scoring system, the organizers decided to use a Borda Count based on the placings for each event to decide on the winner. the places of the athletes in each event are shown below, which athlete was the winner?

\# EVENTS 5

	F	S	Sh	E	CC	AVE.
Achiles Bolt	4	2	5	4	1	16/5
Jordan Michaels	2	1	3	5	2	13/5 *
Inigo Montoya	1	3	4	3	3	14/5
John Wayne	3	4	2	1	5	15/5
Annie Oakley	5	5	1	2	4	17/5

The winner using the Borda Method is:

(a) Inigo Montoya

(b) Achiles Bolt

(c) John Wayne

~~(d)~~ Jordan Michaels

(e) Annie Oakley

2.(6 pts.) There are 5000 tennis players in Racquetland. The number of Left handed tennis players in Racquetland is 1,500, the number of right handed players is 3,700 and the number of ambidextrous players is 300. What is the relative frequency of right handed players in Racquetland?

(a) 0.3 (b) 3.7 ~~(c)~~ 0.74 (d) 3,700 (e) 0.7

REL FREQ = $\dfrac{\text{\# R. HANDED PLAYERS}}{\text{TOTAL \# PLAYERS}}$

$= \dfrac{3700}{5000} = .74$

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3.(6 pts.) There were five candidates for the role for president for the Notre Dame Bubble Football Club. Each of the 20 club members filled out a ballot ranking their preferences for the candidates (1 for their top choice). The results of the election are shown below.

Number of Voters = 20

	3	**3**	**2**	**4**	**5**	**3**
T. Drump	1	5	5	5	1	3
C. Hilton	5	2	4	4	2	2
S. Banders	4	1	3	3	3	1
R. Mubio	2	4	1	2	4	4
C. Truz	3	3	2	1	5	5

HEAD TO HEAD COMP.

T.D. vs S.B. → S.B.
8 vs 12

S.B. vs C.H. → S.B
15 vs. 5

S.B vs R.M. → S.B.
11 vs. 9

S.B. vs. C.T → S.B.
11 vs 9

If a Condorcet winner exists, he/she will win the election. Otherwise a Condorcet completion process is used to decide the winner. Which of the following is true?

(a) C. Hilton is the Condorcet winner

(b) T. Drump is the Condorcet winner

(c) There is no Condorcet winner

(d) R. Mubio and C. Truz are both Condorcet winners

(e) S. Banders is the Condorcet winner

4.(6 pts.) An experiment consists of flipping a coin until a tail appears. As soon as a tail appears, the experimenter stops and the experimenter records the sequence of heads and tails. What is the the probability that the experiment will stop after 4 flips of the coin?

(a) 1/8 (b) 1/16 (c) 8 (d) 16 (e) 1/2

$$P(HHHT) = \frac{1}{2^4} = \frac{1}{16}$$

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5.(6 pts.) If I flip a fair coin 500 times in a row, which of the following numbers is closest to the longest run of Heads you would expect to see in the sequence of outcomes, based on probability?

~~(a)~~ 8 (b) 11 (c) 4 (d) 15 (e) 6

LONGEST RUN OF HEADS IN THE DATA SHOULD BE APPROXIMATELY:

$$\frac{\ln\left(\frac{500}{2}\right)}{\ln(2)} = \frac{\ln(250)}{\ln(2)} = 7.96578 \approx 8$$

6.(6 pts.) Consider the following matrices:

$$A = \begin{pmatrix} 2 & 1 & -1 \\ 3 & 2 & 1 \\ 0 & -2 & 2 \end{pmatrix} \quad B = \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$$

Which of the following matrices is equal to AB?

(a) $\begin{pmatrix} 4 & 7 & -2 \end{pmatrix}$. (b) $\begin{pmatrix} 4 \\ 7 \\ -2 \end{pmatrix}$. ~~(c)~~ $\begin{pmatrix} 6 \\ 7 \\ -4 \end{pmatrix}$

(d) $\begin{pmatrix} 6 & 7 & -4 \end{pmatrix}$. (e) $\begin{pmatrix} 6 \end{pmatrix}$.

$A_{3\times3} \cdot B_{3\times1}$ is a 3×1 MATRIX so ONLY TWO of The ABOVE ANSWERS could BE CORRECT. We FIND The ENTRIES OF $A\cdot B$ by MULTIPLYING THE ROWS OF A by B

$$\begin{pmatrix} 2 & 1 & -1 \end{pmatrix}\begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} = 4+1+1 = 6$$

$$\begin{pmatrix} 3 & 2 & 1 \end{pmatrix}\begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} = 6+2-1 = 7$$

$$\begin{pmatrix} 0 & -2 & 2 \end{pmatrix}\begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} = 0-2-2 = -4$$

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7.(6 pts.) The entries in the following table shows the number of times the row team played the column team for the teams in New Zealand premier league soccer (up to Feb. 20 2016) along with wins minus losses (W-L) and the goal differential for each team (GD).

Abbreviations Aukland City (AC), Team Wellingon (TW), Hawkes Bay (HB) Canterbury United (CU), WaiBOP United, Waitakere United (WU), Wellington Phoenix (WP), Southern United (SU).

GAMES PLAYED		AC	TW	HB	CU	WaiBOP	WU	WP	SU	W-L	GD	$1+\frac{1}{2}(W-L)$
12	AC		2	2	1	1	2	2	2	10	26	6
12	TW	2		1	2	2	2	1	2	5	12	3.5
12	HB	2	1		2	2	1	2	2	5	10	3.5
11	CU	1	2	2		2	1	2	1	4	6	3
13	WaiBOP	1	2	2	2		2	2	2	-1	5	$1/2$
12	WU	2	2	1	1	2		2	2	-4	-12	-1
13	WP	2	1	2	2	2	2		2	-8	-17	-3
13	SU	2	2	2	1	2	2	2		-11	-30	$-9/2$

Which of the matrix equations on the following page must be solved in order to find the Colley Ratings (keeping the same ordering of the teams as above)?

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(a)
$$\begin{pmatrix} 14 & -2 & -2 & -1 & -1 & -2 & -2 & -2 \\ -2 & 14 & -1 & -2 & -2 & -2 & -1 & -2 \\ -2 & -1 & 14 & -2 & -2 & -1 & -2 & -2 \\ -1 & -2 & -2 & 13 & -2 & -1 & -2 & -1 \\ -1 & -2 & -2 & -2 & 15 & -2 & -2 & -2 \\ -2 & -2 & -1 & -1 & -2 & 14 & -2 & -2 \\ -2 & -1 & -2 & -2 & -2 & -2 & 15 & -2 \\ -2 & -2 & -2 & -1 & -2 & -2 & -2 & 15 \end{pmatrix} \begin{pmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \\ r_5 \\ r_6 \\ r_7 \\ r_8 \end{pmatrix} = \begin{pmatrix} 26 \\ 12 \\ 10 \\ 6 \\ 5 \\ -12 \\ -17 \\ -30 \end{pmatrix}$$

(b)
$$\begin{pmatrix} 12 & -2 & -2 & -1 & -1 & -2 & -2 & -2 \\ -2 & 12 & -1 & -2 & -2 & -2 & -1 & -2 \\ -2 & -1 & 12 & -2 & -2 & -1 & -2 & -2 \\ -1 & -2 & -2 & 11 & -2 & -1 & -2 & -1 \\ -1 & -2 & -2 & -2 & 13 & -2 & -2 & -2 \\ -2 & -2 & -1 & -1 & -2 & 12 & -2 & -2 \\ -2 & -1 & -2 & -2 & -2 & -2 & 13 & -2 \\ -2 & -2 & -2 & -1 & -2 & -2 & -2 & 13 \end{pmatrix} \begin{pmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \\ r_5 \\ r_6 \\ r_7 \\ r_8 \end{pmatrix} = \begin{pmatrix} 6 \\ 7/2 \\ 7/2 \\ 3 \\ 1/2 \\ -1 \\ -3 \\ -9/2 \end{pmatrix}$$

(c)
$$\begin{pmatrix} 14 & -2 & -2 & -1 & -1 & -2 & -2 & -2 \\ -2 & 14 & -1 & -2 & -2 & -2 & -1 & -2 \\ -2 & -1 & 14 & -2 & -2 & -1 & -2 & -2 \\ -1 & -2 & -2 & 13 & -2 & -1 & -2 & -1 \\ -1 & -2 & -2 & -2 & 15 & -2 & -2 & -2 \\ -2 & -2 & -1 & -1 & -2 & 14 & -2 & -2 \\ -2 & -1 & -2 & -2 & -2 & -2 & 15 & -2 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \\ r_5 \\ r_6 \\ r_7 \\ r_8 \end{pmatrix} = \begin{pmatrix} 6 \\ 7/2 \\ 7/2 \\ 3 \\ 1/2 \\ -1 \\ -3 \\ 0 \end{pmatrix}$$

(d)
$$\begin{pmatrix} 14 & -2 & -2 & -1 & -1 & -2 & -2 & -2 \\ -2 & 14 & -1 & -2 & -2 & -2 & -1 & -2 \\ -2 & -1 & 14 & -2 & -2 & -1 & -2 & -2 \\ -1 & -2 & -2 & 14 & -2 & -1 & -2 & -1 \\ -1 & -2 & -2 & -2 & 14 & -2 & -2 & -2 \\ -2 & -2 & -1 & -1 & -2 & 14 & -2 & -2 \\ -2 & -1 & -2 & -2 & -2 & -2 & 14 & -2 \\ -2 & -2 & -2 & -1 & -2 & -2 & -2 & 14 \end{pmatrix} \begin{pmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \\ r_5 \\ r_6 \\ r_7 \\ r_8 \end{pmatrix} = \begin{pmatrix} 6 \\ 7/2 \\ 7/2 \\ 3 \\ 1/2 \\ -1 \\ -3 \\ -9/2 \end{pmatrix}$$

~~(e)~~
$$\begin{pmatrix} 14 & -2 & -2 & -1 & -1 & -2 & -2 & -2 \\ -2 & 14 & -1 & -2 & -2 & -2 & -1 & -2 \\ -2 & -1 & 14 & -2 & -2 & -1 & -2 & -2 \\ -1 & -2 & -2 & 13 & -2 & -1 & -2 & -1 \\ -1 & -2 & -2 & -2 & 15 & -2 & -2 & -2 \\ -2 & -2 & -1 & -1 & -2 & 14 & -2 & -2 \\ -2 & -1 & -2 & -2 & -2 & -2 & 15 & -2 \\ -2 & -2 & -2 & -1 & -2 & -2 & -2 & 15 \end{pmatrix} \begin{pmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \\ r_5 \\ r_6 \\ r_7 \\ r_8 \end{pmatrix} = \begin{pmatrix} 6 \\ 7/2 \\ 7/2 \\ 3 \\ 1/2 \\ -1 \\ -3 \\ -9/2 \end{pmatrix}$$

$\leftarrow 1 + \frac{1}{2}(W - L)$

$-$# games played by team i and team j

2 + # games played

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Partial Credit

You must show your work on the partial credit problems to receive credit!

8.(4 pts.) Match the properties of voting systems shown on the left below with their definitions shown on the right by drawing a line connecting the property to its definition.

Independence from irrelevant alternatives

Non-Dictatorship

Universal Domain

Pareto Optimality

Any ordering of the candidates is allowed

If all voters prefer candidate A to candidate B, then the group choice should not prefer candidate B to candidate A.

No one individual voter preference totally determines the group choice.

If a group of voters choose candidate A over candidate B, then the addition or subtraction of other candidates should not change the group choice to B.

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9.(6 pts.) Eight Players, listed as Player 1-8 below, will play in a round robin tournament where each player playes every other player exactly once. Make out a schedule for the tournament in the matrix below by inserting the number of the player that Player i will play in Round j in row i and column j.

	Round 1	**Round 2**	**Round 3**	**Round 4**	**Round 5**	**Round 6**	**Round 7**
Player 1	8	2	3	4	5	6	7
Player 2	7	1	8	3	4	5	6
Player 3	6	7	1	2	8	4	5
Player 4	5	6	7	1	2	3	8
Player 5	4	8	6	7	1	2	3
Player 6	3	4	5	8	7	1	2
Player 7	2	3	4	5	6	8	1
Player 8	1	5	2	6	3	7	4

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10.(20 pts.) A Marathon Runner passes a refreshment stand every five miles. The stands stock two types of beverages containing electrolytes, Crocade and Powerfuel.

- Crocade contains 25 mg (milligrams) of sodium per fluid ounce and 8 mg of potassium per fluid ounce.
- The Powerfuel contains 15 mg of sodium per fluid ounce and 12 mg of potassium per fluid ounce.
- The runner wants to make sure she gets exactly 950 mg of sodium and 520 mg of potassium during the race.

Let x denote the number of fluid ounces of Crocade that the runner consumes during the race and let y denote the number of fluid ounces of Powerfuel they consume during the race.

(a) Write down the set of linear equations in x and y that must be solved to find the amounts of each drink which should be consumed during the race to fulfill the requirement.

	Croc. (x)	Pow. (y)	Wanted
Sod.	25	15	950
Pot.	8	12	520

$$\begin{aligned} 25\text{x} + 15\text{ y} &= 950 \\ 8\text{x} + 12\text{y} &= 520 \end{aligned}$$

(b) Convert the linear equations from part (a) into a matrix equation of the form $AX = B$ and write the matrix equation below.

$$\begin{bmatrix} 25 & 15 \\ 8 & 12 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 950 \\ 520 \end{bmatrix}$$

(c) Find the determinant and the inverse of the matrix A from part (b).

$$\det(A) = (25 \times 12) - (8 \times 15) = 180$$

$$A^{-1} = \frac{1}{180} \begin{bmatrix} 12 & -15 \\ -8 & 25 \end{bmatrix}$$

(d) Find $A^{-1}B$ and solve for the number of fluid ounces of each type of drink the runner should consume during the race.

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{180} \begin{bmatrix} 12 & -15 \\ -8 & 25 \end{bmatrix} \begin{bmatrix} 950 \\ 520 \end{bmatrix} = \frac{1}{180} \begin{bmatrix} 3600 \\ 5400 \end{bmatrix} = \begin{bmatrix} 20 \\ 30 \end{bmatrix}$$

Crocade _20_ ounces, Powerfuel _30_ ounces.

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11.(10 pts.) This problem appears as Problem 1 on the take home part of the exam. You may use this page for rough work.

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12.(18 pts.) This problem appears as Problem 2 on the take home part of the exam. You may use this page for rough work.

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Instructor: ANSWERS

Math 10170, Exam I
March 1, 2016

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PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1.	(a)	(b)	(c)	(•)	(e)
2.	(a)	(b)	(•)	(d)	(e)
3.	(a)	(b)	(c)	(d)	(•)
4.	(a)	(•)	(c)	(d)	(e)
5.	(•)	(b)	(c)	(d)	(e)
6.	(a)	(b)	(•)	(d)	(e)
7.	(a)	(b)	(c)	(d)	(•)

Please do NOT write in this box.

Multiple Choice	______
8.	______
9.	______
10.	______
11.	______
12.	______
Total	______