Name: ____________________________

Instructor: ________________________

Math 10170, Exam II
April 11, 2016

- The Honor Code is in effect for this examination. All work is to be your own.
- You may use your Calculator.
- The exam lasts for 50 minutes.
- Be sure that your name is on every page in case pages become detached.
- Be sure that you have all 11 pages of the test.

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1.	(a)	(b)	(c)	(d)	(e)
2.	(a)	(b)	(c)	(d)	(e)
3.	(a)	(b)	(c)	(d)	(e)
4.	(a)	(b)	(c)	(d)	(e)
5.	(a)	(b)	(c)	(d)	(e)
6.	(a)	(b)	(c)	(d)	(e)
7.	(a)	(b)	(c)	(d)	(e)

Please do NOT write in this box.

Multiple Choice	______________
8.	______________
9.	______________
10.	______________
11.	______________
Total	______________

Name: ____________________________

Instructor: _________________________

Multiple Choice

1.(6 pts.) In a dice game a player pays one dollar to play.

- The attendant first rolls a fair four sided die.
- The attendant then rolls a fair six sided die.
- If the sum of the numbers on the uppermost faces of the dice is greater than 8, the attendant gives the player $2, Let A be the event that the sum is > 8
- if not the player gets no money back. Let B be the event that the # on the 4 sided die is > 1

If the attendant rolls a number larger than 1 on the four sided die, which of the answers shown below gives the probability that the sum of the two numbers will be greater than 8, given that the number on the first roll was larger than 1? Q: What is $P(A|B)$?

Note: that the outcomes for the experiment can be listed as an equally likely sample space

B →

{(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)}

$A = A \cap B$

~~(a)~~ $\frac{1}{6}$ (b) $\frac{1}{8}$ (c) $\frac{3}{4}$ (d) $\frac{1}{3}$ (e) $\frac{1}{12}$

$$P(A|B) = \frac{\# A \cap B}{\# B} = \frac{3}{18} = \frac{1}{6}$$

2.(6 pts.) "Long drive" is a competitive sport where success is derived by hitting a golf ball the farthest by driving. The probability that John will drive the ball more than 375 feet on any given attempt is 0.7. If John attempts 500 drives in a row, what is the expected length of the longest run of drives over 375 feet? Choose the answer closest to yours.

$p = 0.7$ $K = 500$

(a) 5 (b) 26 (c) 7 ~~(d)~~ 14 (e) 20

Expected length of longest run of success in $K = 500$ drives =

$$-\frac{\ln((1-p)500)}{\ln(p)} = -\frac{\ln(0.3 \times 500)}{\ln(0.3)} \simeq 14$$

Name: ______________________________

Instructor: __________________________

3.(6 pts.) The following table shows the probability distribution of the random variable X. Find $E(X)$ (the expected value of X).

k (x)	Pr(X = k) ($P(x)$)	$xP(x)$
-1	1/2	$-1/2$
0	1/8	0
1	1/8	$1/8$
2	1/4	$1/2$
	SUM	$1/8 = E(x)$

(a) $-1/8$ (b) 1 ~~(c)~~ 1/8 (d) 1/2 (e) 3/2

4.(6 pts.) The following table shows the probability distribution of the random variable X. The expected value of X is $E(X) = 1$ (there is no need to check this), find $\sigma(X)$, the standard deviation of X.

$\mu = 1$

k (x)	Pr(X = k) ($P(x)$)	$x-\mu$	$(x-\mu)^2$	$(x-\mu)^2P(x)$
-1	1/4	-2	4	1
0	1/8	-1	1	$1/8$
1	1/4	0	0	0
2	1/8	1	1	$1/8$
3	1/4	2	4	1
		SUM =		$9/4 = \sigma^2$

$\sigma = \sigma(X)$

$= \sqrt{\sigma^2}$

$= \sqrt{9/4}$

$= 3/2$

(a) 7/5 (b) 1 (c) 9/4 ~~(d)~~ 3/2 (e) $\sqrt{3/4}$

Name: ______________________________

Instructor: ___________________________

5.(6 pts.) Coach McEvil has each of his football players perform a cone drill at the start of practice on Saturday mornings. The drill requires the student to move around 3 cones. At the end of the drill, the player must do 5 pushups for every cone he knocked over during the drill. Matt has a probability of 0 of knocking over the first cone in the drill, the probability he will knock over the second cone is 0.3 and the probability he will knock over the third cone is 0.5. The event that Matt knocks down any particular cone is independent of his performance on any other cone. Let X be the random variable which is equal to the number of pushups Matt will have to do after this drill. Which of the following gives the probability distribution for X?
Note: A Tree diagram might help.

(a)

k	Pr(X = k)
0	0.15
5	0.5
10	0.35

(b)

k	Pr(X = k)
0	0.25
5	0.5
10	0.25

(c)

k	Pr(X = k)
0	0.3
5	0.5
10	0.2

(d)

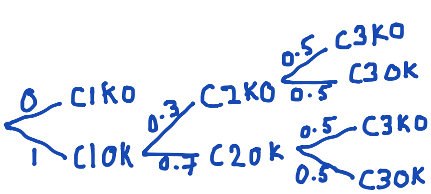
k	Pr(X = k)
0	0.35
5	0.5
10	0.15

(e)

k	Pr(X = k)
0	0.35
5	0.35
10	0.15
15	0.15

The possible # of cones that Matt can knock over is 0, 1 or 2. Therefore, the possible values for X = # push ups Matt will do is 0, 5 or 10.

$P(X=0)$ = P(Matt does not knock over any cones) = product of probabilities that he does not knock over cone 1, 2 and 3 respectively (by independence).
$= 1\cdot(0.7)(0.5) = 0.35.$
$P(X=10)$ = Prob Matt knocks over cones 2 and 3 (since he never knocks over cone 1).
$= (0.3)(0.5) = 0.35$
Now since this is a prob. dist., we must have $P(X=0)+P(X=5)+P(X=10)=1$.
Therefore $P(X=5)=1-0.35-0.15 = 1-0.5=0.5$.



Name: ______________________________

Instructor: ______________________________

6.(6 pts.) Two beer companies Catwater and Ratwater are considering running an advertisement during the next super bowl (R) or not running such an advertisement (NR). Both companies will make their decisions independently without knowledge of the decision of the other company. The cost of running an advertisement is \$5 million dollars. Since the beers made by each company are very similar, the revenue gained from such an advertisement for each company depends on whether the other company runs an ad during the super bowl or not.

- If both companies run an advertisement during the super bowl, the revenue for Catwater will increase by \$4.5 million and the revenue for Ratwater will increase by \$4 million.
- If only one company runs an advertisement during the super bowl, the revenue for that company will increase by \$10 million and the revenue for the other company will decrease by \$1.5 million.
- If neither company advertises during the super bowl, the revenue of both companies will remain unchanged.

Recall that profit is revenue minus cost. Which of the following shows the payoff matrix for this simultaneous move game, where the pay-offs shown (in millions of dollars) are the change in profit for both companies in each scenario.

(a)

		Catwater R	NR
Ratw.	R	(-1,-0.5)	(5,-1.5)
	NR	(-1.5,5)	(0, 0)

(b)

		Catwater R	NR
Ratw.	R	(4.5, 4)	(10, 5)
	NR	(5, 10)	(5, 5)

(c)

		Catwater R	NR
Ratw.	R	(4.5, 4)	(10, 5)
	NR	(5, 10)	(0, 0)

(d)

		Catwater R	NR
Ratw.	R	(4.5, 4)	(10, -1.5)
	NR	(-1.5, 10)	(0, 0)

(e)

		Catwater R	NR
Ratw.	R	(-0.5, -1)	(5, -6.5)
	NR	(-6.5, 5)	(-5, -5)

CATWATER

STRATEGY Rw	Cw	INC. COST	INC REV.	INC Profit
R	R	5	4.5	-0.5
R	NR	0	-1.5	-1.5
NR	R	5	10	5
NR	NR	0	0	0

RATWATER

STRATEGY Rw	Cw	INC COST	INC. REV.	INC. Profit
R	R	5	4	-1
R	NR	5	10	5
NR	R	0	-1.5	-1.5
NR	NR	0	0	0

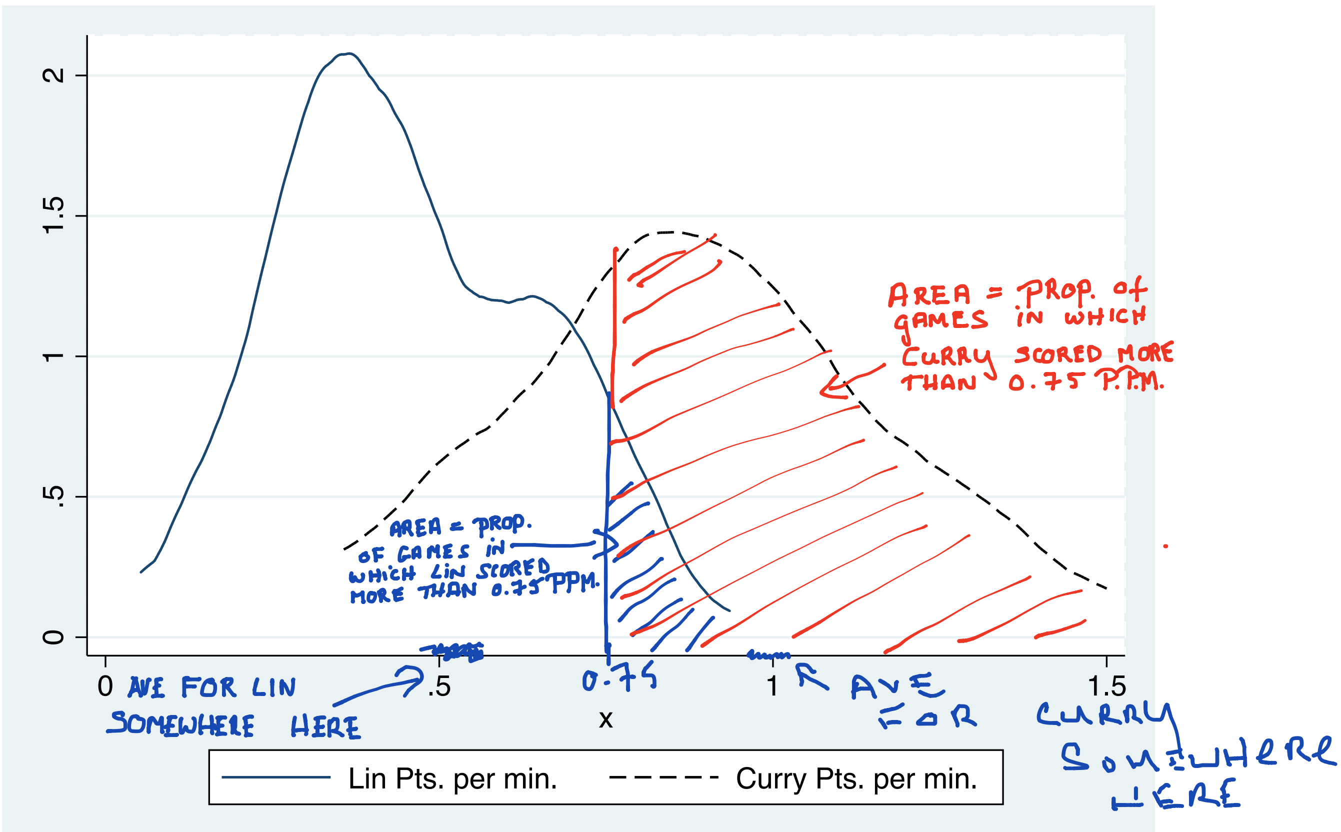
CAT.

RAT.	R	NR
R	(-1, -0.5)	(5, -1.5)
NR	(-1.5, 5)	(0,0)

Name: ____________________________

Instructor: ________________________

7.(6 pts.) The following graphs shows the distribution of the random variable X = "Points per minute of play (PPM)" for games in the 2015-2016 regular season for basketball players Jeremy Lin and Steven Curry.



Which of the following statements is true?

(a) S. Curry's average PPM per game in the 2015-2016 regular season was less than 0.6 FALSE

(b) J. Lin scored more than 0.75 PPM in more than half of his games in the 2015-2016 regular season. FALSE

(c) S. Curry scored more than 0.75 PPM in more than half of his games in the 2015-2016 regular season. TRUE

(d) J. Lin's average PPM per game in the 2015-2016 regular season was greater than 0.6 FALSE

(e) The average PPM per game in the 2015-2016 regular season was the same for S. Curry and J. Lin FALSE

Name: ____________________________

Instructor: ________________________

Partial Credit

You must show your work on the partial credit problems to receive credit!

8.(4 pts.) The following data shows whether a basketball player made a basket (B) or missed (M) on 40 consecutive three point shots:

$$M\ M\ B\ B\ B\ M\ M\ B\ B\ B\ B\ B\ B\ M\ M\ B\ M\ B\ B\ B$$
$$M\ B\ B\ B\ B\ M\ M\ B\ M\ M\ B\ B\ B\ M\ M\ B\ M\ B\ B\ M$$

(a) How many runs (of B's and M's) are in the data?

19 RUNS

(b) If X denotes the number of runs in a randomly generated sequence of B's and M's of length N where the number of B's is N_B and the number of M's is N_M, then X has an approximately normal distribution with mean

$$E(X) = \frac{2(16)(24)}{40} + 1 = 20.2$$

$$\mu = E(X) = \frac{2N_M N_B}{N} + 1 \text{ and } \sigma(X) = \sqrt{\frac{(\mu-1)(\mu-2)}{N-1}}.$$

$$\sigma(X) = \sqrt{\frac{(19.2)(18.2)}{39}} \approx 2.99333$$

Applying this to the above set of data, fill in the values of the parameters in the table below

N	N_B	N_M	$\mu = E(X)$	σ
40	24	16	20.2	2.99333

(c) What is the Z-score of the number of runs observed in the above set of data?

$$Z = \frac{X - \mu}{\sigma} = \frac{19 - 20.2}{2.99333} \approx -0.400895$$

(d) If you were testing the Null hypothesis that the above data was randomly generated against the alternative hypothesis that it was not randomly generated at a 5% level of significance, what would your conclusion be?

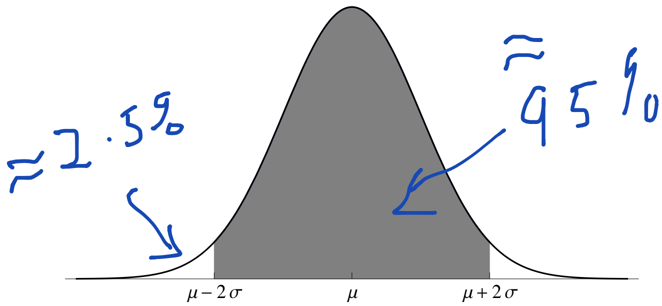
REJECT H_0 if. $|Z| > 2$

WE DO NOT REJECT H_0.

Name: ______________________________

Instructor: __________________________

9.(20 pts.) The picture below shows a normal density curve (mound shaped distribution) for a random variable X with mean μ and standard deviation σ.



(a) Roughly what percentage of observations of X lie in the shaded region?

95 %

(b) Using statistics collected over a number of years, the mean and standard deviations of times for wide receivers have been estimated for several events in the NFL combine (times shown are in seconds):

Event	**WR mean**	**WR St. deviation**
40 yard dash	4.51	0.1
20 yard shuttle	4.26	0.14
3 cone drill	6.96	0.2

Assuming that the distributions of these variables are mound shaped, roughly what percentage of wide receivers have a time for the 40 yard dash less than 4.31?

$4.31 = 4.51 - 2(0.1) = \mu - 2\sigma$ ANS: 2.5%

(c) In the table below we show the times for these three events for Will Fuller in the 2016 NFL combine. Find Will's Z-score for each event and fill in the table below.

Event	**40 yard dash:**	**20 yard shuttle:**	**3 cone drill:**
Will's Time	4.32 seconds	4.27 seconds	6.93 seconds
Will's Z-Score	−1.9	0.07	−0.15

$\frac{6.93 - 6.96}{0.2}$

$\frac{4.27 - 4.26}{0.14}$

$\frac{4.32 - 4.51}{0.1}$

(d) In which event did Will have the most outstanding performance?

$\boxed{\text{The 40 yd. dASH.}}$

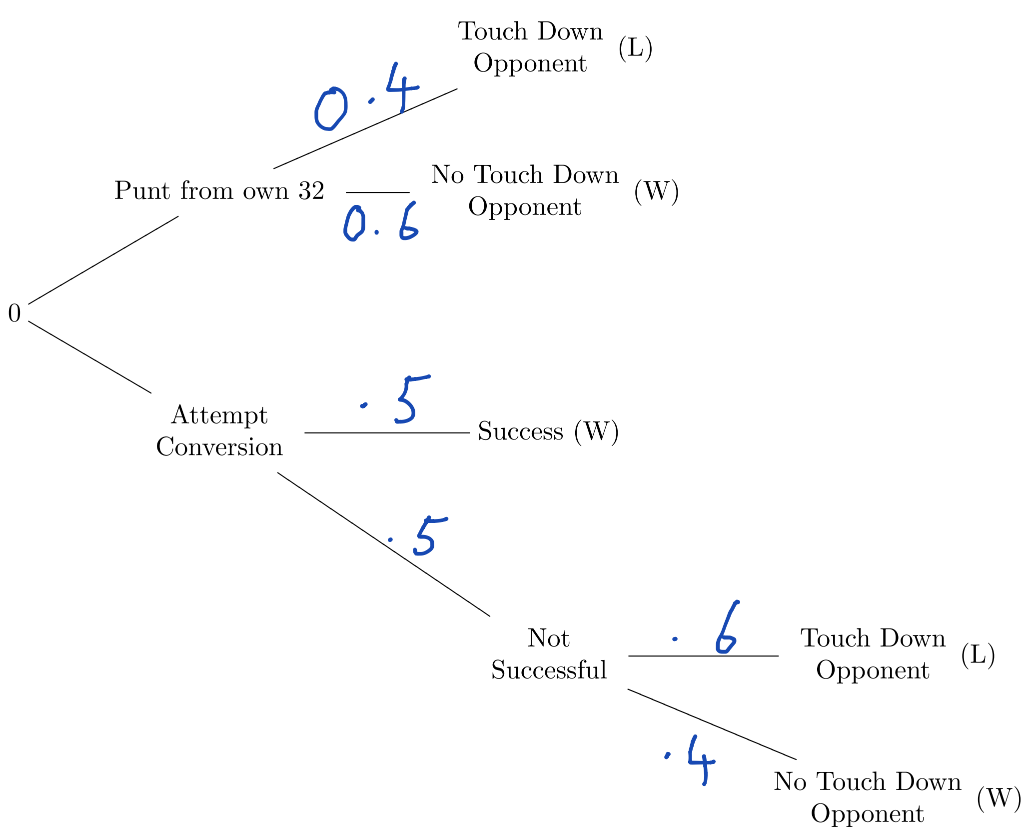
Name: ______________________________

Instructor: __________________________

10.(10 pts.) In American Football, when a team gains possession of the ball, they have 4 chances to advance at least 10 yards. If the team does not advance at least 10 yards, they lose the ball after the fourth attempt. Each play is called a down and on fourth down, the coach is faced with making a decision on whether to punt the ball or attempt a conversion. Coach McEvil is faced with such a decision. His team has the ball at their own 32 yard line and are up by 6 points with 2 minutes left on the clock.

- If they punt the ball from this position, Coach McEvil figures that the probability that the opposing team will return the ball for a touchdown (and win the game) is 40%.
- If they attempt a conversion from this position, there is a fifty percent chance that it will be successful and
- if the conversion is not successful there is a 60% chance that the opponent will return the ball for a touchdown and win the game.

(a) Fill in the probabilities according to Coach McEvil's assessment on the tree diagram below.



(b) Calculate the probability that Coach McEvil's team will win (W) if they attempt a conversion.

Note: Parts (c) and (d) are on the next page. $(.5)+(.5)(.4)=.7$

Name: ______________________________

Instructor: __________________________

Question 10 continued

(c) Calculate the probability that Coach McEvil's team will win (W) if they punt.

0.6

(d) Which strategy should Coach McEvil choose, Punt or attempt a conversion?

ATTEMPT A CONVERSION

Name: ____________________________

Instructor: ________________________

11.(20 pts.) This is just a place holder for your grade for the questions you will hand in as the take home part of your exam. You may use this page for rough work.