Name: __________________________
Instructor: __________________________

Math 10550, Self Diagnostic Exam
August 28, 2014

• The Honor Code is in effect for this examination. All work is to be your own.
• No calculators.
• The exam lasts for 1 hour .
• Be sure that your name is on every page in case pages become detached.
• Be sure that you have all 15 pages of the test.

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1.  (a)  (b)  (c)  (d)  (e)
2.  (a)  (b)  (c)  (d)  (e)
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15. (a)  (b)  (c)  (d)  (e)
16. (a)  (b)  (c)  (d)  (e)
17. (a)  (b)  (c)  (d)  (e)
Multiple Choice

1. (1 pt) Which of the following expressions is equal to
\[
\left( \frac{2x^{1/4}}{y^{1/5}} \right)^5 \left( \frac{y^2}{x} \right)
\]
\[
\left( \frac{2x^{1/4}}{y^{1/5}} \right)^5 \left( \frac{y^2}{x} \right) = \left( \frac{2^5 x^{5/4}}{y^{5/5}} \right) \left( \frac{y^2}{x} \right)
\]
\[
= 32x^{5/4-1}y^{2-1} = 32x^{1/4}y.
\]
If you did not get the correct answer to this question please review Lecture 1 under the link [Algebra/Precalculus Review](#) on our website.

(a) \( \frac{2x}{y^{1/5}} \)  \hspace{1cm} (b) \( 32xy \)  \hspace{1cm} (c) \( 32x^{1/4}y \)  \hspace{1cm} (d) \( 2y^{-1} \)  \hspace{1cm} (e) \( \frac{32x}{y^{3/5}} \)

2. (1 pt) Which of the following numbers is equal to
\[
\sqrt[3]{-8} + \left| \frac{7 - 10}{21} \right|
\]
where \( |x| \) denotes the absolute value of the number \( x \).
\[
\sqrt[3]{-8} + \left| \frac{7 - 10}{21} \right| = -2 + \left| \frac{-3}{21} \right| = -2 + \left| \frac{-1}{7} \right|
\]
\[
= -2 + \frac{1}{7} = -\frac{13}{7}.
\]
If you did not get the correct answer to this question please review Lectures 1 and 2 under the link [Algebra/Precalculus Review](#) on our website.

(a) \( -\frac{15}{7} \)  \hspace{1cm} (b) \( \frac{13}{7} \)  \hspace{1cm} (c) \( \frac{15}{7} \)  \hspace{1cm} (d) \( -\frac{13}{7} \)  \hspace{1cm} (e) 5
3. (1 pt) Which of the following expressions is equal to 

\[(x - 1)(x + 1) + (\sqrt{x + 1} - 1)^2\]

\[(x - 1)(x + 1) + (\sqrt{x + 1} - 1)^2 = x^2 - 1 + (x + 1) - 2\sqrt{x + 1} + 1 = x^2 + x - 2\sqrt{x + 1} + 1\]

If you did not get the correct answer to this question please review Lecture 3 under the link [Algebra/Precalculus Review](#) on our website.

(a) \(x^2 + x - 1\)  
(b) \(x^2 + x - 2\sqrt{x + 1} + 1\)

(c) \(x^2 + x + 1\)  
(d) \(x^2 + 2\sqrt{x + 1} + 1\)

(e) \(x^2 + x + 3\)

4. (1 pt) Which of the following expressions is equal to

\[\frac{1}{x^2 - 16} + \frac{x}{x + 4}\]

\[
\frac{1}{x^2 - 9} + \frac{x}{x + 4} = \frac{(x - 4)(x + 4)}{(x - 3)(x + 3)} + \frac{x}{x + 4} = \frac{1}{(x - 3)(x + 3)} + \frac{x}{x + 4}
\]

\[
= \frac{1}{(x + 3)(x + 4)} + \frac{x}{x + 4} = \frac{1 + x(x + 3)}{(x + 3)(x + 4)} = \frac{1 + x^2 + 3x}{(x + 3)(x + 4)}
\]

If you did not get the correct answer to this question please review Lecture 4 under the link [Algebra/Precalculus Review](#) on our website.

(a) \(\frac{x^3 - 8x^2 + 17x + 3}{(x + 4)(x - 4)^2}\)  
(b) \(\frac{x + 1}{(x + 4)(x + 3)}\)  
(c) \(\frac{x^2 + 3x + 1}{(x + 4)(x + 3)}\)

(d) \(\frac{x + 1}{(x + 4)^2(x + 3)}\)  
(e) \(\frac{x - 1}{(x + 4)^2(x + 3)}\)
5. (1 pt) Which of the following expressions is equal to
\[
\frac{1}{a + h} - \frac{1}{a}
\]
asuming that \(a\) is a real number with \(a \neq h\) and \(h\) is a real number with \(h \neq 0\).
\[
\frac{1}{a + h} - \frac{1}{a} = \frac{1}{h} \left( \frac{a - (a + h)}{(a + h)(a)} \right) \cdot \frac{1}{h} = \frac{-h}{(a + h)(a)} \cdot \frac{1}{h} = \frac{-1}{(a + h)(a)}
\]
If you did not get the correct answer to this question please review Lecture 4 under the link Algebra/Precalculus Review on our website

(a) \(\frac{-h^2}{a(a + h)}\) (b) \(\frac{1}{a(a + h)}\) (c) \(\frac{-1}{(a + h)}\)

(d) \(\frac{h}{a(a + h)}\) (e) \(\frac{-1}{a(a + h)}\)

6. (1 pt) A ball thrown vertically upwards from a height of 96 feet has height above the ground at time \(t\) (seconds after being thrown) given by the function \(h(t)\) shown below.
\[
h(t) = -16t^2 + 16t + 96.
\]
When does the ball first hit the ground?
When the ball hits the ground, it has height equal to 0, that is \(h(t) = 0\).
\[
h(t) = 0 \text{ if } -16t^2 + 16t + 96 = 0 \text{ if } -16(t^2 - t - 6) = 0
\]
if \(t^2 - t - 6 = 0\) if \((t - 3)(t + 2) = 0\) if \(t = 3\) or \(t = -2\).
Now since \(t\) denotes time, \(t = -2\) is impossible, so the ball first hits the ground when \(t = 3\).
If you did not get the correct answer to this question please review Lecture 5 under the link Algebra/Precalculus Review on our website

(a) at \(t = 3\) seconds (b) the ball never hits the ground.
(c) at \(t = 16\) seconds (d) at \(t = 6\) seconds
(e) at \(t = 2\) seconds
7. (1 pt) What is the domain of the following function

\[ f(x) = \frac{1}{x-1} + \frac{x}{(x+2)\sqrt{2x+1}} \]

We must have non-zero denominators and we note that the square root can only be applied to numbers \( \geq 0 \). Therefore we must have \( x \neq 1, x \neq -2, 2x + 1 \neq 0 \) and \( 2x + 1 > 0 \).

We have \( 2x + 1 > 0 \) if \( x > -\frac{1}{2} \) and since we must also have \( x \neq 1 \), our domain is \((-\frac{1}{2}, 1) \cup (1, \infty)\).

If you did not get the correct answer to this question please review Lectures 7 and 10 under the link Algebra/Precalculus Review on our website.

(a) \((-\infty, -2) \cup (-2, -\frac{1}{2}) \cup (-\frac{1}{2}, 1) \cup (1, \infty)\)

(b) \([-\frac{1}{2}, 1) \cup (1, \infty)\)

(c) all real numbers.

(d) \([\frac{1}{2}, 1) \cup (1, 2) \cup (2, \infty)\)

(e) \((-\frac{1}{2}, 1) \cup (1, \infty)\)
8. (1 pt) Two cylindrical swimming pools are being filled simultaneously with water being pumped into each at the same rate of $10 m^3/min$. (Both pools were empty to start with and the water started pumping into both pools at the same time.) The smaller pool has a radius of 5 m. and the larger pool has a radius of 8 m. What is the height of the water in the larger pool when the height of the water in the smaller pool is 1 meter?

We let $V_1(t)$ denote the volume of water in the smaller pool after $t$ minutes and we let $V_2(t)$ denote the volume of water in the larger pool after $t$ minutes. Because water is pumped into both pools at the same constant rate of $10 m^3/min$, (both starting from empty at the same time) we have

$$V_1(t) = V_2(t) = 10t m^3.$$ 

Let $h_1(t)$ denote the height of the water in the smaller pool after $t$ minutes and let $h_2(t)$ denote the height of the water in the larger pool after $t$ minutes. We have

$$V_1(t) = 10t = \pi 5^2 h_1(t).$$

Therefore

$$t = \frac{25\pi h_1(t)}{10}.$$ 

When $h_1(t) = 1$, we have $t = \frac{25\pi}{10}$. We also have $V_2(t) = 10t = \pi 8^2 h_2(t)$. Therefore

$$h_2(t) = \frac{10t}{64\pi}.$$ 

When $h_1(t) = 1$ or $t = \frac{25\pi}{10}$, we have

$$h_2(t) = \frac{10 \cdot 25\pi}{64\pi \cdot 10} = \frac{25}{64} \text{ meters}.$$ 

If you did not get the correct answer to this question please review Lecture 6 under the link [Algebra/Precalculus Review](#) on our website.

(a) $\frac{8\pi}{5}$ meters (b) $\frac{64\pi}{25}$ meters (c) $\frac{64}{25}$ meters (d) $\frac{25}{64}$ meters (e) $\frac{5}{8}$ meters
9. (1 pt) Solve the inequality \( x^2 + 3 \leq 4x \)

Rearranging the above inequality, we get \( x^2 - 4x + 3 \leq 0 \).
Factoring, we get \((x - 3)(x - 1) \leq 0\).
The expression on the left can only change sign at 3 and 1.
We check the sign of each factor on each interval:

\[
\begin{array}{|c|c|c|c|}
\hline
\text{Interval} & (-\infty, 1) & (1, 3) & (3, \infty) \\
\hline
\text{Sign of } x - 1 & - & + & + \\
\hline
\text{Sign of } x - 3 & - & - & + \\
\hline
\text{Sign of } (x - 1)(x - 3) & + & - & + \\
\hline
\end{array}
\]

We get \((x - 3)(x - 1) \leq 0\) for \(1 \leq x \leq 3\).

If you did not get the correct answer to this question please review Lecture 7 under the link Algebra/Precalculus Review on our website.

(a) True for all real numbers \(x\).  
(b) \(1 \leq x \leq 3\)

(c) \(x \in (-\infty, 1) \cup (3, \infty)\)  
(d) \(1 < x < 3\)

(e) \(x \in (-\infty, 1] \cup [3, \infty)\)

10. (1 pt) Find the equation of the line that passes through the points \((0, 1)\) and \((3, -1)\).

The slope of the line is given by \(m = \frac{-1 - 1}{3 - 0} = -\frac{2}{3}\). The equation of a line through \((x_1, y_1)\) with slope \(m\) is given by \(y - y_1 = m(x - x_1)\). With \((x_1, y_1) = (0, 1)\), we get the equation of our line is given by

\[y - 1 = -\frac{2}{3}x.\]

If you did not get the correct answer to this question please review Lecture 9 under the link Algebra/Precalculus Review on our website.

(a) \(y = -4x + 1\)  
(b) \(y = -\frac{2}{3}x - \frac{1}{3}\)  
(c) \(y = -\frac{2}{3}x + 1\)

(d) \(y = 4x + 1\)  
(e) \(y = \frac{2}{3}x + 1\)
11. (1 pt) Consider the function

\[ g(x) = \begin{cases} 
  x + 1 & \text{if } x > 1 \\
  x^2 & \text{if } -1 \leq x \leq 1 \\
  4 & \text{if } x < -1 
\end{cases} \]

Which of the following is true?

- \( g(1/2) = (1/2)^2 = 1/4 \) since \(-1 < 1/2 < 1\).
- \( g(-1/2) = (-1/2)^2 = 1/4 \) since \(-1 < -1/2 < 1\).
- \( g(-1) = (-1)^2 = 1 \) since \(-1 \in [-1, 1]\).
- \( g(-5) = 4 \) since \(-5 < -1\).
- \( g(5) = 5 + 1 = 6 \) since \(5 > 1\).

Therefore the answer is

\[ g(1/2) = 1/4, \quad g(-1) = 1, \quad g(-5) = g(5) - 2. \]

If you did not get the correct answer to this question please review Lecture 10 under the link [Algebra/Precalculus Review](#) on our website

(a) \( g(1/2) = g(-1/2), \quad g(-1) = 0, \quad g(-5) = g(5) + 1 \)
(b) \( g(1/2) = 1/4, \quad g(-1) = 4, \quad g(-5) = g(5) - 1 \)
(c) \( g(1/2) = 1/4, \quad g(-1) = 1, \quad g(-5) = g(5) - 2 \)
(d) \( g(1/2) = 1/4, \quad g(-1) = 1, \quad g(-5) = g(5) \)
(e) \( g(1/2) = 3/4, \quad g(-1) = 4, \quad g(-5) = 6 \)
12. (1 pt) Consider the function \( f(x) = x^2 + 1 \). Let \( h \) be a real number with \( h \neq 0 \). Which of the following is equal to 

\[
\frac{f(\pi + h) - f(\pi)}{h}
\]

\[
\frac{(\pi + h)^2 + 1 - (\pi^2 + 1)}{h} = \frac{\pi^2 + 2h\pi + h^2 + 1 - \pi^2 - 1}{h}
\]

\[
= \frac{2h\pi + h^2}{h} = h(2\pi + h) = 2\pi + h
\]

when \( h \neq 0 \).

If you did not get the correct answer to this question please review Lecture 10 under the link Algebra/Precalculus Review on our website.

(a) \( \frac{2\pi^2 + 2\pi + 2}{h} \) 
(b) \( \frac{2\pi + 1}{h} \) 
(c) \( 2\pi + 1 \)

(d) \( 2\pi + h \) 
(e) \( \pi + h \)
13. (1 pt) Let \( f(x) = x^2 + 1 \) and \( g(x) = \frac{1}{x-2} \). Which of the following is equal to \((g \circ f)(2)\)?

\[
(g \circ f)(2) = g(f(2)) = g(4 + 1) = g(5) = \frac{1}{5 - 1} = 1/3.
\]

If you did not get the correct answer to this question please review Lecture 13 under the link Algebra/Precalculus Review on our website.

(a) 1/2 (b) 1/3 (c) 5/3 (d) 2 is not in the domain of \( g \circ f \).

(e) 0
14. (1 pt) Which of the following graphs give the graph of a function?

Only A and D pass the Vertical Line test, so only A and D are graphs of functions.
If you did not get the correct answer to this question please review Lecture 11 under
the link Algebra/Precalculus Review on our website.
(a) A, B, C and D are all graphs of functions.
(b) A only
(c) A and D only
(d) A, C and D only
(e) None of the above graphs give the graph of a function
15. (1 pt) Which of the following is the graph of the function 
\[ \sin(x + \frac{\pi}{2}) + 1 \]

The graph below is what we get when we shift the graph of \( y = \sin x \) to the left by \( \pi/2 \) units and upwards by one unit.

Therefore this is the graph of \( y = \sin(x + \frac{\pi}{2}) + 1 \).

If you did not get the correct answer to this question please review Lectures 12 and 14 under the link **Algebra/Precalculus Review** on our website.
16. (1 pt) Evaluate the following

\[
\frac{\sin \left(\frac{\pi}{2}\right) + \sin \left(\frac{5\pi}{6}\right)}{\cos \left(\frac{5\pi}{4}\right)}
\]

\[
\sin \left(\frac{\pi}{2}\right) = 1, \quad \sin \left(\frac{5\pi}{6}\right) = \frac{1}{2}, \quad \cos \left(\frac{5\pi}{4}\right) = -1/\sqrt{2}.
\]

Therefore

\[
\frac{\sin \left(\frac{\pi}{2}\right) + \sin \left(\frac{5\pi}{6}\right)}{\cos \left(\frac{5\pi}{4}\right)} = \frac{1 + \frac{1}{2}}{-\frac{1}{\sqrt{2}}} = -\frac{3}{2} \cdot \frac{\sqrt{2}}{1} = -\frac{3}{\sqrt{2}}
\]

If you did not get the correct answer to this question please review Lecture 14 under the link Algebra/Precalculus Review on our website.

(a) \(-\frac{3}{\sqrt{2}}\)  (b) \(-\frac{2+\sqrt{3}}{\sqrt{2}}\)  (c) \(-\frac{1}{\sqrt{2}}\)  (d) \(\frac{1}{\sqrt{2}}\)  (e) \(\frac{3}{\sqrt{2}}\)
17. (1 pt) Which of the following expressions is equal to
\[
\frac{\sin^2 x + \cos^2 x}{\tan x}
\]
Since \(\sin^2 x + \cos^2 x = 1\) and \(\tan x = \frac{\sin x}{\cos x}\), we have
\[
\frac{\sin^2 x + \cos^2 x}{\tan x} = \frac{1}{\sin x} = \frac{\cos x}{\sin x}
\]
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