Differentiation Formulas

As we did with limits and continuity, we will introduce several properties of the derivative and use them along with the derivatives of some basic functions to make calculation of derivatives easier.

**Constant Functions and Power Functions**

**Derivative of a Constant**: \( \frac{d}{dx}(c) = 0 \), if \( c \) is a constant.

**Power Rule**: If \( n \) is a positive integer, then \( \frac{d}{dx}(x^n) = nx^{n-1} \).

**Example** If \( g(x) = 2 \), \( f(x) = x^3 \), find \( f'(x) \) and \( g'(x) \).

Just as with limits, we have the following rules:

**Constant Multiple Rule**: \( \frac{d}{dx}[cf(x)] = c \frac{d}{dx}f(x) \), where \( c \) is a constant and \( f \) is a differentiable function.

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\end{align*} \]
**Sums and Differences**

**The Sum Rule** if \(f\) and \(g\) are both differentiable at \(x\), then \(f + g\) is differentiable at \(x\) and

\[
\frac{d}{dx} [f(x) + g(x)] = \frac{d}{dx} f(x) + \frac{d}{dx} g(x)
\]

**The Difference Rule** if \(f\) and \(g\) are both differentiable at \(x\), then \(f - g\) is differentiable at \(x\) and

\[
\frac{d}{dx} [f(x) - g(x)] = \frac{d}{dx} f(x) - \frac{d}{dx} g(x)
\]

**Example** Find the derivative of the function \(f(x) = x^2 + 2x + 4\).

**Example** Find the derivative of the function \(f_1(x) = x^{12} - 10x^6 + 3x + 1\).
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\[ f'(x) = \frac{d}{dx}x^2 + \frac{d}{dx}(2x) + \frac{d}{dx}4 = 2x + 2 + 0 = 2x + 2. \]

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$\Rightarrow f_1'(x) = \frac{d}{dx}x^{12} - \frac{d}{dx}(10x^6) + \frac{d}{dx}(3x) + \frac{d}{dx}1 = 12x^{11} - 60x^5 + 3 + 0$
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\frac{d}{dx}[f(x)g(x)] = f(x)\frac{d}{dx}[g(x)] + g(x)\frac{d}{dx}[f(x)].
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This can be rewritten in a number of ways

$$
\frac{d}{dx}(uv) = v \frac{du}{dx} + u \frac{dv}{dx}, \quad \text{or} \quad (fg)' = gf' + fg'.
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**Example**  Let $k(x) = x(x^2 + 2x + 4)$, find $k'(x)$.

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  = (2t^3 + t^2)(2t) + (t^2 + 4)(6t^2 + 2t)
\]
Let $g$ be differentiable and non-zero at $x$, then
\[
\frac{d}{dx} \left( \frac{1}{g(x)} \right) = -\frac{g'(x)}{(g(x))^2}
\]

**Example** The curve $y = 1/(1 + x^2)$ is called a **Witch of Maria Agnesi**. Find the equation of the tangent line to the curve at the point $(-1, 1/2)$. 

\[q(x) = \frac{1}{1 + x^2}\]
**Special Case of The quotient Rule**

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According to the above rule,

\[
\frac{dy}{dx} = \frac{-g'(x)}{(g(x))^2} = \frac{-2x}{(1+x^2)^2}.
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When $x = -1$, $\frac{dy}{dx} \bigg|_{x=-1} = \frac{2}{(1+1)^2} = \frac{2}{4} = \frac{1}{2}$.
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When $x = -1$, $\left. \frac{dy}{dx} \right|_{x=-1} = \frac{2}{(1+1)^2} = \frac{1}{2}$.

Since $(-1, 1/2)$ is a point on the tangent, we have the equation of the tangent to the curve at $x = -1$ is given by

$$y - 1/2 = \frac{1}{2}(x + 1) \quad \text{or} \quad y = \frac{1}{2}x + \frac{1}{2} = \frac{1}{2}$$

or $$y = \frac{1}{2}x + 1.$$
The Quotient Rule

We can combine the above rules to get the quotient rule:
If $f$ and $g$ are differentiable at $x$ and $g(x) \neq 0$, then $\frac{f}{g}$ is differentiable at $x$ and

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\frac{d}{dx} \left[ \frac{f(x)}{g(x)} \right] = \frac{g(x) \frac{d}{dx} [f(x)] - f(x) \frac{d}{dx} [g(x)]}{[g(x)]^2}.
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\frac{d}{dx} \left( \frac{u}{v} \right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2} \quad \text{or} \quad \left( \frac{f}{g} \right)' = \frac{gf' - fg'}{g^2}.
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- Let \( f(x) = x^3 + x^2 + 1 \) and \( g(x) = x^4 + 1 \).
- The rule above says

\[
K'(x) = \frac{g(x) \frac{d}{dx} [f(x)] - f(x) \frac{d}{dx} [g(x)]}{[g(x)]^2} = \frac{(x^4 + 1) \frac{d}{dx} [x^3 + x^2 + 1] - (x^3 + x^2 + 1) \frac{d}{dx} (x^4 + 1)}{[(x^4 + 1)]^2}
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The Quotient Rule

We can combine the above rules to get the quotient rule:
If \( f \) and \( g \) are differentiable at \( x \) and \( g(x) \neq 0 \), then \( \frac{f}{g} \) is differentiable at \( x \) and

\[
\frac{d}{dx} \left[ \frac{f(x)}{g(x)} \right] = \frac{g(x) \frac{d}{dx} [f(x)] - f(x) \frac{d}{dx} [g(x)]}{[g(x)]^2}.
\]

We can rewrite this as

\[
\frac{d}{dx} \left( \frac{u}{v} \right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2} \quad \text{or} \quad \left( \frac{f}{g} \right)' = \frac{gf' - fg'}{g^2}.
\]

Example  Let \( K(x) = \frac{x^3+x^2+1}{x^4+1} \), find \( K'(x) \). What is \( K'(1) \)?

\begin{itemize}
  \item let \( f(x) = x^3 + x^2 + 1 \) and \( g(x) = x^4 + 1 \).
  \item The rule above says
  \[
  K'(x) = \frac{g(x) \frac{d}{dx} [f(x)] - f(x) \frac{d}{dx} [g(x)]}{[g(x)]^2} = \frac{(x^4+1) \frac{d}{dx} [x^3+x^2+1] - (x^3+x^2+1) \frac{d}{dx} (x^4+1)}{[(x^4+1)]^2}
  \]
  \[
  = \frac{(x^4+1)(3x^2+2x)-(x^3+x^2+1)4x^3}{[(x^4+1)]^2}
  \]
\end{itemize}
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  \]
  \[
  K'(1) = \frac{(2)(5)-(3)^4}{[(2)]^2} = \frac{-2}{4} = \frac{-1}{2}
  \]
Example

Here’s a way to remember the quotient rule:
"low d-high minus hi d-low, square the bottom and away we go”
or
Low D High minus High D Low, all over the square of what’s below.

Note we should see if we simplify a function with cancellation before we rush into using the quotient rule.

Example Find the derivative of \( L(x) = \frac{x^6 + x^4 + x^2}{x^2} \).
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L'(x) &= 4x^3 + 2x.
\end{align*}
\]
General Power Functions

When $n$ is a positive integer $x^n = x \cdot x \cdot x \cdot \cdots \cdot x$, where the product is taken $n$ times. We define $x^0 = 1$ and $x^{-n} = 1/x^n$. We can use the quotient rule to show that. If $n$ is a positive integer

$$\frac{d}{dx}(x^{-n}) = -nx^{-n-1}$$

In fact it can be shown that if $k$ is any real number

$$\frac{d}{dx}(x^k) = kx^{k-1}.$$  

Example  If $H(x) = 2/x^2 + 3/x^3 + 4/x^4 + 1$ find $H'(x)$.

Example  Find the derivative of $f(x) = \frac{\sqrt{x}+x^2+x^{1/3}}{x\sqrt{2}}$.  

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According to the power rule, \( H'(x) = -4x^{-3} - 9x^{-4} - 16x^{-5} \)

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\[
f'(x) = (\frac{1}{2} - \sqrt{2})x^{(-\frac{1}{2} - \sqrt{2})} + (2 - \sqrt{2})x^{(1 - \sqrt{2})} + (\frac{1}{3} - \sqrt{2})x^{(-\frac{2}{3} - \sqrt{2})}
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