Name: ______________________________

Instructor: __________________________

Math 10550, Exam III
November 15, 2011

- The Honor Code is in effect for this examination. All work is to be your own.
- No calculators.
- The exam lasts for 1 hour and 15 min.
- Be sure that your name is on every page in case pages become detached.
- Be sure that you have all 10 pages of the test.

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1.	(a)	(b)	(c)	(d)	(e)
2.	(a)	(b)	(c)	(d)	(e)
3.	(a)	(b)	(c)	(d)	(e)
4.	(a)	(b)	(c)	(d)	(e)
5.	(a)	(b)	(c)	(d)	(e)
6.	(a)	(b)	(c)	(d)	(e)
7.	(a)	(b)	(c)	(d)	(e)
8.	(a)	(b)	(c)	(d)	(e)
9.	(a)	(b)	(c)	(d)	(e)
10.	(a)	(b)	(c)	(d)	(e)

Please do NOT write in this box.

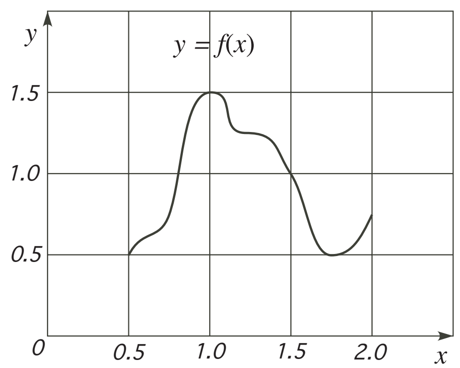
Multiple Choice	________
11.	________
12.	________
13.	________
14.	________
Total	________

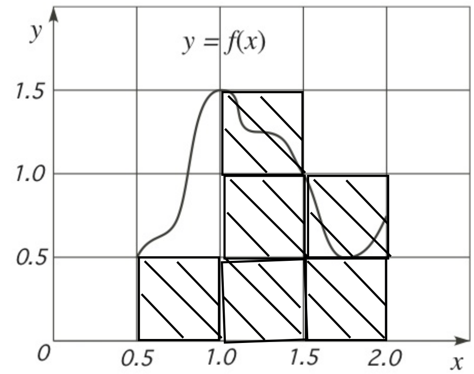
Name: ______________________________

Instructor: __________________________

Multiple Choice

1.(6 pts.) Estimate the area under the graph of $y = f(x)$ between $x = 0.5$ and $x = 2.0$, using a Riemann sum with **three** equal subintervals, using the **left-hand** endpoints.





The left end point approximation with $n = 3$ is the area of the shaded region above, which is $6 \times (0.5)^2 = 1.25$.

(a) 3.0 (b) 2.0 (c) 1.5 (d) 1.0 (e) 4.0

2.(6 pts.) Find $\lim_{n\to\infty} \frac{n(n+1)(2n+1)}{6n^3}$.

$$\lim_{n\to\infty} \frac{n(n+1)(2n+1)}{6n^3} = \frac{1}{6} \lim_{n\to\infty} \frac{(n+1)(2n+1)}{n^2} = \frac{1}{6} \lim_{n\to\infty} \frac{(n+1)}{n} \lim_{n\to\infty} \frac{(2n+1)}{n} = \frac{2}{6} = \frac{1}{3}.$$

(a) ∞ (b) 1 (c) 1/6 (d) 0 (e) 1/3

Name: ____________________________

Instructor: ________________________

3.(6 pts.) Find the equation of the slant asymptote as $x \to \infty$ of the function

$$f(x) = \frac{2x^3}{x^2 - 1}.$$

Using long division, we see that $\dfrac{2x^3}{x^2-1} = 2x + \dfrac{2x}{x^2-1}$.

Therefore $\lim_{x\to\infty}[f(x) - 2x] = \lim_{x\to\infty} \dfrac{2x}{x^2-1} = 0$.

Therefore $y = 2x$ is a slant asymptote to the graph of the function $f(x)$.

(a) $y = 0$ (b) $y = 2x$ (c) $y = -x + 2$

(d) $y = -2x$ (e) $y = 2x + 1$

4.(6 pts.) In finding an approximate solution to the equation $x^4 - 2x^3 - 5 = 0$ using Newton's method with initial approximation $x_1 = 2$, what is x_2?

We have

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 2 - \frac{f(2)}{f'(2)}.$$

$f'(x) = 4x^3 - 6x^2$ and $f'(2) = 4(2)^3 - 6(2)^2 = 32 - 24 = 8$.

Also $f(2) = 2^4 - 2(2^3) - 5 = 16 - 16 - 5 = -5$.

Therefore

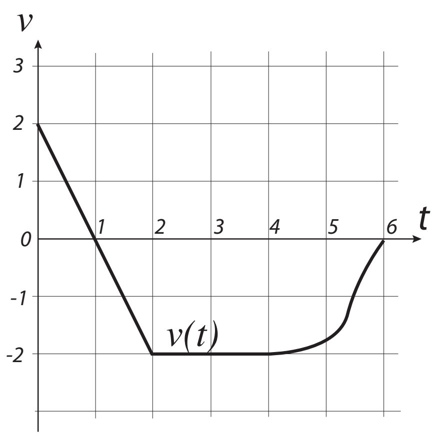
$$x_2 = 2 - \frac{(-5)}{8} = 2 + \frac{5}{8} = \boxed{\frac{21}{8}}.$$

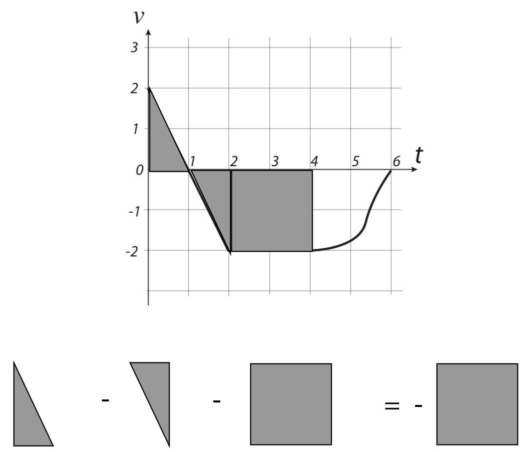
(a) 5/8 (b) 18/5 (c) 2/5 (d) 21/8 (e) 11/8

Name: ______________________________

Instructor: __________________________

5.(6 pts.) The graph of the function $v(t)$ is given below:





Find $\int_0^4 v(t)\,dt$.

The definite integral $\int_0^4 v(t)\,dt$ is the net signed area of the graph between 0 and 4. From the picture, we can see that the net signed area is $-$(area of rectangle shown above) $= -4$.

(a) -4 (b) 0 (c) 4 (d) -2 (e) 2

6.(6 pts.) Find an antiderivative $F(x)$ of $f(x) = 2x + 3\sqrt{x}$ satisfying $F(1) = 4$. Which of the following is $F(4)$?

$$F(x) = \frac{2x^2}{2} + 3\frac{x^{3/2}}{3/2} + C = x^2 + 2x^{3/2} + C.$$

$F(1) = 4$ implies that $1^2 + 2(1)^{3/2} + C = 4$ which implies that $1 + 2 + C = 4$, which implies that $C = 1$.

Therefore $F(x) = x^2 + 2x^{3/2} + 1$ and $F(4) = 4^2 + 2(4)^{3/2} + 1 = 16 + 2(8) + 1 = \boxed{33}$.

(a) 16 (b) 9 (c) 27 (d) 33 (e) 7

Name: ____________________________

Instructor: _________________________

7.(6 pts.) Evaluate the Riemann sum for $f(x) = 2 - x^2$ with $0 \leq x \leq 2$, using **four** subintervals and taking the sample points to be the **right-hand** endpoints of the intervals.

We use 4 subintervals, with length $\Delta x = \dfrac{2-0}{4} = 1/2$ and endpoints

$$x_0 = 0, \quad x_1 = 1/2, \quad x_2 = 1, \quad x_3 = 3/2, \quad x_4 = 2.$$

We make a table of values of function values at the endpoints:

x_i	0	1/2	1	3/2	2
$f(x_i) = 2 - 2x_i^2$	2	7/4	1	−1/4	−2

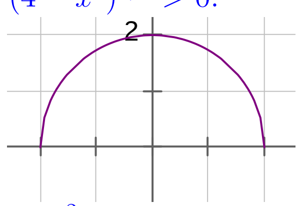
The value of the Riemann sum using the right end points with $n = 4$ is

$$R_4 = f(x_1)\Delta x + f(x_2)\Delta x + f(x_3)\Delta x + f(x_4)\Delta x$$
$$= \Delta x[f(x_1) + f(x_2) + f(x_3) + f(x_4)]$$
$$= 1/2[7/4 + 1 - 1/4 - 2] = 1/2[2/4] = 1/4$$

(a) 0.2 (b) 1.5 (c) 2.5 (d) 0.25 (e) 0.36

8.(6 pts.) By interpreting the integral as an area, evaluate $\int_{-2}^{2} (4 - x^2)^{1/2}\, dx$.

The graph of $y = (4 - x^2)^{1/2}$ is a semicircle of radius 2 as shown below. (since squaring both sides gives $y^2 = 4 - x^2$ and $(4 - x^2)^{1/2} > 0$.



The value of the definite integral $\int_{-2}^{2} (4 - x^2)^{1/2}\, dx$ is the area under this curve between -2 and 2. The area under the curve between -2 and 2 is one half of the area of a circle with radius 2. Therefore we get

$$\int_{-2}^{2} (4 - x^2)^{1/2}\, dx = \frac{1}{2}\pi 4 = 2\pi.$$

(a) $\dfrac{\sqrt{2}}{2}\pi$ (b) 2π (c) 4π (d) π (e) 0

Name: ______________________________

Instructor: __________________________

9.(6 pts.) Evaluate $\displaystyle\int_1^2 \frac{x^2+\sqrt{x}}{x}\,dx$.

$$\int_1^2 \frac{x^2+\sqrt{x}}{x}\,dx = \int_1^2 x + x^{-1/2}\,dx = \frac{x^2}{2} + x^{1/2}1/2\Big|_1^2$$

$$= \frac{4}{2} + 2\sqrt{2} - [\frac{1}{2}+2] = \boxed{2\sqrt{2}-\frac{1}{2}}.$$

(a) $2 - \dfrac{\sqrt{2}}{2}$ (b) $3\sqrt{2}$ (c) $2\sqrt{2} + \dfrac{1}{2}$

(d) $\sqrt{2}+2$ (e) $2\sqrt{2} - \dfrac{1}{2}$

10.(6 pts.) If $F(x) = \displaystyle\int_{x^2}^4 (t+1)\,dt$, find $F'(x)$.

We use the chain rule:

$$F'(x) = \frac{d}{dx}\int_{x^2}^4 (t+1)\,dt = \frac{d}{d(x^2)}\int_{x^2}^4 (t+1)\,dt\frac{dx^2}{dx} = -(x^2+1)2x = -2x(x^2+1).$$

(a) $-2x^3 - 2x$ (b) $2x^3 + 2x$ (c) x^2+1

(d) x (e) $\dfrac{x^2}{2} + x$

Name: ______________________________

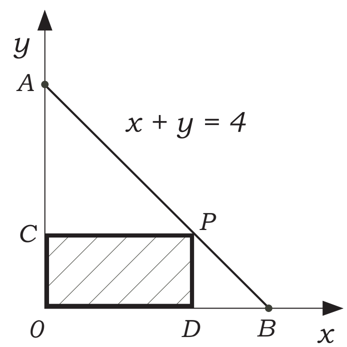
Instructor: __________________________

Partial Credit

You must show your work on the partial credit problems to receive credit!

11.(10 pts.) A rectangle $CODP$ (with sides CP and OD parallel to the x-axis) is inscribed in the region bounded by the lines $x + y = 4$ and the coordinate axes, with the corner P being on the line segment AB (including possibly at A or at B).

(a) Write the area $A(x)$ of the rectangle in terms of x, the x-coordinate of P.



We have $|OD| = x$ and $|DP| = 4 - x$, where $|OD|$ and $|DP|$ denote the length of the corresponding line segments. Therefore $A(x) = x(4 - x) = 4x - x^2$.

(b) What is the range of possible values of x?

Since we allow A and B as corners of the rectangle, x may take any value from the interval $[0, 4]$.

(c) Find the value of x that maximizes the area $A(x)$. For full credit, you must show that your answer **maximizes** $A(x)$.

We must find the maximum of the continuous function $A(x)$ on the closed interval $[0, 4]$. We know that the maximum occurs either at a critical point or at the end points.

Critical Points: $A'(x) = 4 - 2x$. We have a critical point where $4 - 2x = 0$ or $\boxed{x = 2}$.

To verify that we have a maximum of the function $A(x)$ at $x = 2$, we compare function values:

$$A(0) = 0, \quad A(2) = 4, \quad A(4) = 0.$$

By comparison, we see that the area is maximized when $\boxed{x = 2}$.

Name: ____________________________

Instructor: ________________________

12.(10 pts.) A ship is sailing along the path $y = 3x + 1$ (with units being nautical miles). A lighthouse is located at the point $(1, 1)$. How close does the ship come to the lighthouse? For full credit, be sure to show why the answer you have found is the **minimum** distance.

Hint: It might be easier to first minimize the **square** of the distance from the ship to the lighthouse.

A point on the ship's path has co-ordinates $(x, y) = (x, 3x + 1)$.
The distance from such a point to the point $(1, 1)$ is given by

$$\begin{aligned} D(x) &= \sqrt{(x-1)^2 + (3x+1-1)^2} \\ &= \sqrt{(x-1)^2 + (3x)^2} \\ &= \sqrt{x^2 - 2x + 1 + 9x^2} \\ &= \sqrt{10x^2 - 2x + 1} \end{aligned}$$

- -

To find the minimum of $D(x)$, we look at the critical points of the function.

$$D'(x) = \frac{1}{2}\frac{20x - 2}{\sqrt{10x^2 - 2x + 1}} = \frac{20x - 2}{2\sqrt{10x^2 - 2x + 1}}.$$

The critical points occur when $D'(x) = 0$ or $D'(x)$ does not exist.

Since $10x^2 - 2x + 1 = (x-1)^2 + (3x+1-1)^2$ has no zeros (because the point $(1, 1)$ is not on the curve $y = 3x + 1$), we can conclude that $D'(x)$ exists everywhere.

$$D'(x) = 0 \text{ if } 20x - 2 = 0 \text{ or } \boxed{x = 1/10.}$$

- -

Since

$$D'(x) = \frac{20(x - 1/10)}{2\sqrt{10x^2 - 2x + 1}}$$

we have $D'(x) > 0$ if $x > 1/10$ and $D'(x) < 0$ if $x < 1/10$. This tells us that $D(x)$ has a global minimum at $x = 1/10$.

- -

When $x = 1/10$, the distance between the ship and the lighthouse is

$$D(1/10) = \sqrt{10/100 - 2/10 + 1} = \sqrt{90/100} = \frac{3\sqrt{10}}{10}.$$

Name: ______________________________

Instructor: __________________________

13.(10 pts.) Let $f(x) = \dfrac{x\sqrt{x^2+1}}{x^2-1}$.

(a) Find the equations of all horizontal asymptotes of $y = f(x)$.

$$\lim_{x\to\infty} \frac{x\sqrt{x^2+1}}{x^2-1} = \lim_{x\to\infty} \frac{(x\sqrt{x^2+1})/x^2}{(x^2-1)/x^2} = \lim_{x\to\infty} \frac{(\sqrt{x^2+1})/x}{1-1/x^2} = \lim_{x\to\infty} \frac{(\sqrt{x^2+1})/\sqrt{x^2}}{1-1/x^2}$$
$$= \lim_{x\to\infty} \frac{\sqrt{1+1/x^2}}{1-1/x^2} = 1$$

$$\lim_{x\to-\infty} \frac{x\sqrt{x^2+1}}{x^2-1} = \lim_{x\to-\infty} \frac{(x\sqrt{x^2+1})/x^2}{(x^2-1)/x^2} = \lim_{x\to-\infty} \frac{(\sqrt{x^2+1})/x}{1-1/x^2} = \lim_{x\to-\infty} \frac{(\sqrt{x^2+1})/(-\sqrt{x^2})}{1-1/x^2}$$
$$= \lim_{x\to-\infty} \frac{-\sqrt{1+1/x^2}}{1-1/x^2} = -1$$

Therefore the lines $y = 1$ and $y = -1$ are horizontal asymptotes to the graph of $y = f(x)$.

(b) Find the equations of all vertical asymptotes of $y = f(x)$.

The lines $x = 1$ and $x = -1$ are vertical asymptotes to the graph of $y = f(x)$, since

$$\lim_{x\to1^+} \frac{x\sqrt{x^2+1}}{x^2-1} = \infty$$

and

$$\lim_{x\to(-1)^+} \frac{x\sqrt{x^2+1}}{x^2-1} = -\infty$$

Name: ______________________________

Instructor: __________________________

14.(10 pts.) A particle is moving along a vertical axis, with the upward direction positive. Its velocity at time $t \geq 0$ (measured in seconds) is $v(t) = 8 - 6t$ (measured in meters per second). Its position at time t is $s(t)$, with $s(0) = 0$.

(a) Find $s(t)$. Find a time $t > 0$ for which $s(t) = 0$.

$$v(t) = 8 - 6t$$
$$s(t) = 8t - \frac{6t^2}{2} + C = 8t - 3t^2 + C$$
$$s(0) = 0 \ \rightarrow \ C = 0$$
$$S(t) = 0 \text{ if } 8t - 3t^2 = 0 \text{ if } t(8 - 3t) = 0 \text{ if } t = 0 \text{ or } \boxed{t = 8/3}.$$

(b) At the time found in part (a), at what speed is the particle moving, and in what direction?

When $t = 8/3$, $v(t) = 8 - \frac{6 \cdot 8}{3} = 8 - 16 = -8.$

The speed at this time is 8 meters per second and the particle is moving downwards.

(c) Find the total distance that the particle travels between $t = 0$ and $t = 1$.

The total distance traveled is given by

$$\int_0^1 |v(t)|\, dt = \int_0^1 |8 - 6t|\, dt$$

Since $8 - 6t > 0$ if $0 \leq t \leq 1$, we have

$$\int_0^1 |8 - 6t|\, dt = \int_0^1 (8 - 6t)\, dt = (8t - 3t^2)\Big|_0^1 = (8 - 3) - 0 = 5 \text{ meters}$$

Name: ____________________

Instructor: ANSWERS

Math 10550, Exam III
November 15, 2011

- The Honor Code is in effect for this examination. All work is to be your own.
- No calculators.
- The exam lasts for 1 hour and 15 min.
- Be sure that your name is on every page in case pages become detached.
- Be sure that you have all 10 pages of the test.

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1.	(a)	(b)	(•)	(d)	(e)
2.	(a)	(b)	(c)	(d)	(•)
3.	(a)	(•)	(c)	(d)	(e)
4.	(a)	(b)	(c)	(•)	(e)
5.	(•)	(b)	(c)	(d)	(e)
6.	(a)	(b)	(c)	(•)	(e)
7.	(a)	(b)	(c)	(•)	(e)
8.	(a)	(•)	(c)	(d)	(e)
9.	(a)	(b)	(c)	(d)	(•)
10.	(•)	(b)	(c)	(d)	(e)

Please do NOT write in this box.

Multiple Choice	______
11.	______
12.	______
13.	______
14.	______
Total	______