1. If \( f(2) = 5, f(3) = 2, f(4) = 5, g(2) = 6, g(3) = 2 \) and \( g(4) = 0 \), find \((f \cdot g)(2) + f(g(3))\).

Solution. \((f \cdot g)(2) + f(g(3)) = f(2) \cdot g(2) + f(2) = 5 \cdot 6 + 5 = 35\).

2. Evaluate the following limit

\[
\lim_{x \to 0} \frac{2 - \sqrt{4 - x^2}}{x^2}.
\]

Solution.

\[
\lim_{x \to 0} \frac{2 - \sqrt{4 - x^2}}{x^2} = \lim_{x \to 0} \frac{2 - \sqrt{4 - x^2}}{x^2} \cdot \frac{2 + \sqrt{4 - x^2}}{2 + \sqrt{4 - x^2}} = \lim_{x \to 0} \frac{4 - (4 - x^2)}{x^2(2 + \sqrt{4 - x^2})} = \lim_{x \to 0} \frac{1}{2 + \sqrt{4 - x^2}} = \frac{1}{4}.
\]

3. For which value of the constant \( c \) is the function \( f(x) \) continuous on \((-\infty, \infty)\)?

\[
f(x) = \begin{cases} 
c^2x - c & x \leq 1 \\
cx - x & x > 1.
\end{cases}
\]

Solution. The partial functions of \( f(x) \) are continuous for \( x < 1 \) and \( x > 1 \) because they are polynomials. To get \( f(x) \) continuous on \((-\infty, \infty)\) we need

\[
\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x) = f(1).
\]

This happens when \( c^2 - c = c - 1 \). Rearranging gives \( 0 = c^2 - 2c + 1 = (c - 1)^2 \), and thus \( c = 1 \).

4. Compute

\[
\lim_{x \to \pi/2^+} \tan x.
\]

Solution. From the graph of \( y = \tan x \), the limit is \(-\infty\). Or, since \( \tan x = \frac{\sin x}{\cos x} \) and \( \sin(\pi/2) = 1 \) and \( \cos(\pi/2) = 0 \), \( \tan x \) has a vertical asymptote at \( x = \pi/2 \). Thus the limit is either \( \infty \) or \(-\infty\). For \( \pi/2 < x < \pi \), we have \( \sin x > 0 \) and \( \cos x < 0 \). Thus for \( x \) near \( \pi/2 \) but greater than \( \pi/2 \), \( \tan x < 0 \). Therefore the answer must be \(-\infty\).
Since the function 
\[ f(x) = \frac{x^2 - 1}{x^3 - 4x} \]
is a rational function, it is continuous everywhere in its domain, which is everywhere that the denominator is nonzero. The denominator is zero at \( x = 0 \) and \( x = \pm 2 \).

6. If \( f(x) = (x^2 + 3x)(6x^5 - 2x^8) \), compute \( f'(1) \).
Solution. \( f'(x) = (2x + 3)(6x^5 - 2x^8) + (x^2 + 3x)(30x^4 - 16x^7) \).
\( f'(1) = 5 \cdot 4 + 4 \cdot 14 = 76 \).

7. For \( f(x) = \sqrt[3]{x} + \frac{6}{\sqrt{x}} \), find \( f'(x) \).
Solution. Rewriting \( f(x) = x^{\frac{2}{3}} + 6x^{-\frac{3}{2}} \), we have \( f'(x) = \frac{5}{3}x^{\frac{1}{3}} + 6\left(-\frac{3}{5}\right)x^{-\frac{5}{2}} = \frac{5\sqrt[3]{x^2}}{3} - \frac{18}{5\sqrt{x^5}} \).

8. Find the equation of the tangent line to 
\[ y = \frac{7x - 3}{6x + 2} \]
at the point \((1, \frac{1}{2})\).
Solution.
\[ y' = \frac{7(6x + 2) - 6(7x - 3)}{(6x + 2)^2} = \frac{32}{(6x + 2)^2} = \frac{8}{(3x + 1)^2} \]
Thus, \( y'(1) = \frac{1}{2} \) which is the slope of the tangent line at \((1, \frac{1}{2})\). Thus \( y = \frac{1}{2}(x - 1) + \frac{1}{2} = \frac{1}{2}x \).

9. If \( f(x) = x^2 \cos x \), find \( f''(x) \).
Solution. Using Product Rule, we get
\[ f'(x) = 2x \cos x - x^2 \sin x, \]
and \( f''(x) = 2 \cos x - 2x \sin x - 2x \sin x - x^2 \cos x \)
\[ = 2 \cos x - 4x \sin x - x^2 \cos x. \]

10. A ball is thrown straight upward from the ground with the initial velocity \( v_0 = 96\text{ft/s} \). Find the highest point reached by the ball. Hint: The height of the ball at time \( t \) is given by \( y(t) = -16t^2 + 96t \).
Solution. Velocity of the ball at time \( t \) is given by
The ball reaches the highest point when $v(t) = 0$, i.e. when $t = 3$ seconds. The height of the ball at 3 seconds is

$$y(3) = -16(3)^2 + 96(3) = -144 + 288 \text{ ft.} = 144 \text{ ft}.$$
Solution.

Let \( f(x) = \frac{1}{x^2 + 1} \).

Then \( y' = f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h} \)

\[
= \lim_{h \to 0} \frac{\frac{1}{(x+h)^2 + 1} - \frac{1}{x^2 + 1}}{h}
\]

\[
= \lim_{h \to 0} \frac{x^2 + 1 - ((x+h)^2 + 1)}{h((x+h)^2 + 1) \cdot (x^2 + 1)} \cdot \frac{1}{h}
\]

\[
= \lim_{h \to 0} \frac{h^2 + x^2 - x^2 - 2xh - h^2 - 1}{h((x+h)^2 + 1)(x^2 + 1)}
\]

\[
= \lim_{h \to 0} \frac{-2x - h}{h((x+h)^2 + 1)(x^2 + 1)}
\]

\[
= \lim_{h \to 0} \frac{-2x - 0}{((x+0)^2 + 1)(x^2 + 1)}
\]

\[
= -\frac{2x}{(x^2 + 1)^2}.
\]