Math 10550, Exam I
September 25, 3024

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- No calculators.
- The exam lasts for 1 hour and 15 min.
- Be sure that your name is on every page in case pages become detached.
- Be sure that you have all 12 pages of the test.
- Each multiple choice question is worth 7 points. Your score will be the sum of the best 10 scores on the multiple choice questions plus your score on questions 13-15.

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1. (a)   (b)   (c)   (d)   (e)

2. (a)   (b)   (c)   (d)   (e)

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3. (a)   (b)   (c)   (d)   (e)

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12. (a)   (b)   (c)   (d)   (e)

Please do NOT write in this box.

Multiple Choice_____

13. _____

14. _____

15. _____

Total _____

Multiple Choice

1.(7 pts.) Compute
$$\lim_{x \to 2} \frac{\sqrt{x^2 + 21} - 5}{x - 2}.$$

(a) $\dfrac{2}{5}$ (b) 0   (c) $\dfrac{1}{10}$ (d) $\dfrac{1}{120}$ (e) 4

2.(7 pts.) Compute
$$\lim_{x \to 0} \frac{\sin(2x)}{\sin(5x)}.$$

(a) $\dfrac{5}{2}$ (b) 0

(c) $\dfrac{2}{5}$ (d) 1

(e) Does not exist.

3.(7 pts.) Compute $\lim\limits_{x \to -1^-} \dfrac{x^2 + x}{x^2 + 2x + 1}$

(a) $-\infty$

(b) -1

(c) 0

(d)   Does not exist and is not ∞ or $-\infty$.

(e) $+\infty$

4.(7 pts.) For what values of c is the function f given by

$$f(x) = \begin{cases} x^2 + c^2 x - 3 & x < 2 \\ cx + 5 & x \geq 2 \end{cases}$$

continuous at $x = 2$?

(a) $c = 1$ only

(b) $c = 2$ and $c = -1$

(c) $c = 0$ only

(d)   No value of c makes f continuous at $x = 2$

(e) $c = 2$ only

5.(7 pts.) Let $f(x) = \sqrt{2x^2 + 1}$. Which of the following limits equals $f'(2)$?

(a) $\displaystyle\lim_{h \to 2} \frac{\sqrt{2(x+h)^2 + 1} - 3}{h}$

(b) $\displaystyle\lim_{x \to 0} \frac{\sqrt{2x^2 + 1} - 3}{x}$

(c) $\displaystyle\lim_{x \to 2} \frac{\sqrt{2x^2 + 1} - 3}{x - 2}$

(d) $\displaystyle\lim_{h \to 2} \frac{\sqrt{2(x+h)^2 + 1} - \sqrt{2x^2 + 1}}{h}$

(e) $\displaystyle\lim_{h \to 0} \frac{\sqrt{2(x+h)^2 + 1} - \sqrt{2x^2 + 1}}{h}$

6.(7 pts.) Assume that $f(x)$ is a continuous function which takes the following values:

x	-1	0	1	2
f(x)	-10	10	-1	3

Which of the following conclusions can we make by using the Intermediate Value Theorem:

(a) $f(x) = 0$ has at most three solutions.

(b) $f(x) = 0$ has exactly three solutions.

(c) $f(x) = 0$ has exactly one solution.

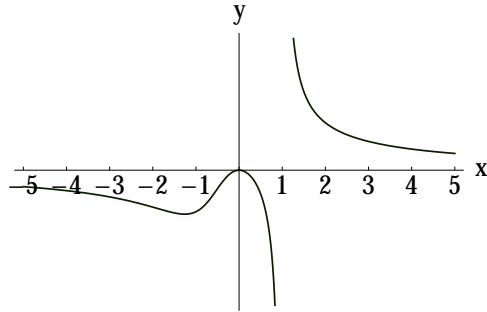(d) $f(x) = 0$ has at least three solutions.

(e) $f(x) = ax^3 + bx^2 + cx + d$ for some constants a, b, c and d.

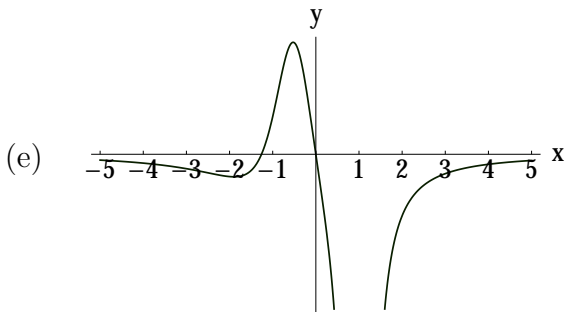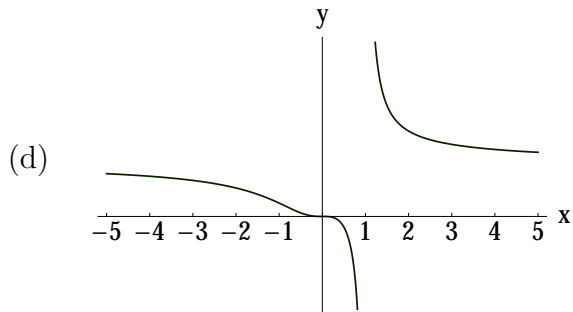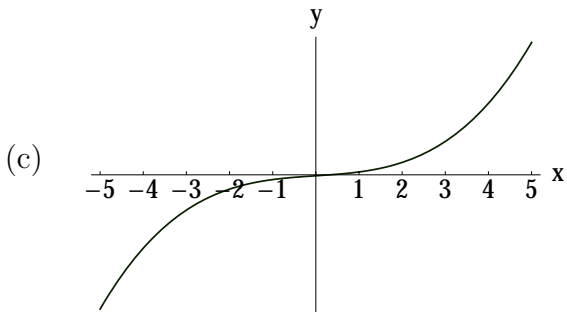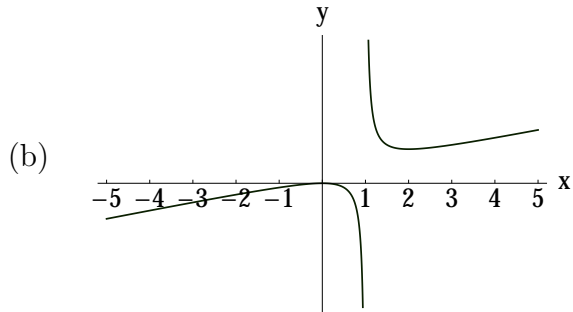**7.**(7 pts.)
 The graph of $f(x)$ is shown below:

 Which of the following is the graph of $f'(x)$?

(a)

(b)

(c)

(d)

(e)
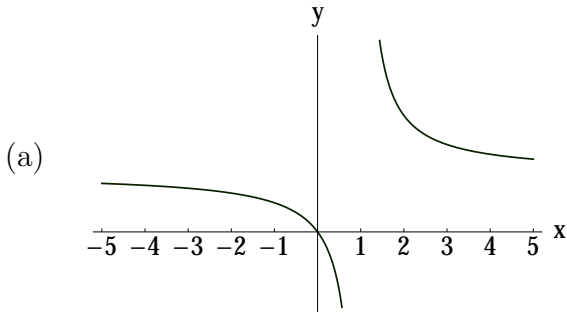
8.(7 pts.) Find $f'(x)$, if

$$f(x) = 2x^2 \sin(\sqrt{x}) + \frac{1}{\sqrt{x}}.$$

(a) $4x\cos(\sqrt{x}) - \dfrac{1}{2\sqrt{x^3}}$

(b) $\sqrt{x^3}\cos(\sqrt{x}) + 4x\sin(\sqrt{x}) - \dfrac{1}{2\sqrt{x^3}}$

(c) $-\sqrt{x^3}\cos(\sqrt{x}) + \sin(\sqrt{x}) + \dfrac{1}{2\sqrt{x^3}}$

(d) $2x^2\cos(\sqrt{x}) + 4x\sin(\sqrt{x}) - \dfrac{1}{2\sqrt{x^3}}$

(e) $-\sqrt{x^3}\cos(\sqrt{x}) + 4x\sin(\sqrt{x}) - \dfrac{1}{2\sqrt{x^3}}$

9.(7 pts.) Find the derivative of $f(x) = \tan(\sin(x^2))$.

(a) $\cot(\sin(x^2))\cos(x^2))$ (b) $-2x\sec^2(\sin(x^2))\cos(x^2))$

(c) $2x\sec^2(\sin(x^2))\cos(x^2))$ (d) $2x\sec^2(\sin(x^2))\sin(x^2)$

(e) $2x\cot(\sin(x^2))\cos(x^2))$

10.(7 pts.) If $f(x) = x\sin x + \cos x$, find $f''(x)$.

(a) $f''(x) = x\cos x + \sin x$

(b) $f''(x) = -x\sin x - \cos x$

(c) $f''(x) = 3\cos x - x\sin x$

(d) $f''(x) = -x\sin x + \cos x$

(e) $f''(x) = -\sin x - \cos x$

11.(7 pts.) Let $h(x) = f \circ g(x) - \dfrac{f(x)}{g(x)}$. If $f(3) = 0$, $g(3) = 1$, $f'(3) = 3$, $g'(3) = 4$, $f'(1) = 7$, and $g'(2) = 5$, then find $h'(3)$.

(a) 30   (b) 20   (c) 25   (d) 10   (e) 0

12.(7 pts.) If $f(x) = x^3 - 3x^2 - 9x + 7$, find the x-coordinates of all points on the curve with horizontal tangent line.

(a) $x = 0$ and $x = 1$

(b) $x = 3$ and $x = -1$

(c) $x = 4$ and $x = -2$

(d) $x = -3$ and $x = 1$

(e) No points on the curve have horizontal tangent line.

Partial Credit
You must show your work on the partial credit problems to receive credit!

13.(13 pts.) Find the derivative of
$$f(x) = \sqrt{x+1}$$
using the limit definition of the derivative.
Please include all of the details in your calculation.

14.(14 pts.) Let $y = x^2 + x$.

(a) Find the equation of the tangent line through the point $(-1, 0)$.

(b) Find all points on the curve whose tangent line goes through the point $(2, 5)$.

15.(13 pts.) Show that there is at least one solution of the equation
$$x^2 = 2 + \sin(\pi x).$$
Justify your answer, identify the theorem you use and explain why the theorem applies.

Name: _____

Instructor: _____

Rough Work

Name: _____

Instructor: _ANSWERS_____

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