Name: ______________________________

Instructor: __________________________

Math 10550, Practice Exam III
November 20, 2024

- The Honor Code is in effect for this examination. All work is to be your own.
- No calculators.
- The exam lasts for 1 hour and 15 min.
- Be sure that your name is on every page in case pages become detached.
- Be sure that you have all 12 pages of the test.
- Each multiple choice question is worth 7 points. Your score will be the sum of the best 10 scores on the multiple choice questions plus your score on questions 13-15.

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1.	(a)	(b)	(c)	(d)	(e)
2.	(a)	(b)	(c)	(d)	(e)
3.	(a)	(b)	(c)	(d)	(e)
4.	(a)	(b)	(c)	(d)	(e)
5.	(a)	(b)	(c)	(d)	(e)
6.	(a)	(b)	(c)	(d)	(e)
7.	(a)	(b)	(c)	(d)	(e)
8.	(a)	(b)	(c)	(d)	(e)
9.	(a)	(b)	(c)	(d)	(e)
10.	(a)	(b)	(c)	(d)	(e)
11.	(a)	(b)	(c)	(d)	(e)
12.	(a)	(b)	(c)	(d)	(e)

Please do NOT write in this box.

Multiple Choice ______________

13. ______________

14. ______________

15. ______________

Total ______________

Name: ____________________________

Instructor: ________________________

Multiple Choice

1.(7 pts.) How many inflection points does the curve $y = \dfrac{x^4}{12} - \dfrac{x^3}{3}$ have?

Solution Notice that

$$y' = \frac{4}{12}x^3 - \frac{3}{3}x^2 = \frac{x^2}{3}(x-3)$$

and

$$y'' = \frac{3}{3}x^2 - 2x = x(x-2).$$

Thus, $y'' = 0$ if and only if $x = 0$ or $x = 2$. Observe that $y'' > 0$ if $x \in (-\infty, 0) \cup (2, \infty)$ and $y'' < 0$ if $x \in (0, 2)$. Given that there are change of sign in $x = 0$ and $x = 2$, we have that both are inflection points. Hence, y has 2 inflection points.

(a) 1 (b) 3 (c) 2 (d) 4 (e) 0

2.(7 pts.) Evaluate $\displaystyle\lim_{x\to-\infty} \frac{3x^3 - 2x + 1}{2x^2 + x + 1}$

Solution

$$\lim_{x\to-\infty} \frac{3x^3 - 2x + 1}{2x^2 + x + 1} = \lim_{x\to-\infty} \frac{\dfrac{3x^3}{x^3} - \dfrac{2x}{x^3} + \dfrac{1}{x^3}}{\dfrac{2x^2}{x^3} + \dfrac{x}{x^3} + \dfrac{1}{x^3}} = \lim_{x\to-\infty} \frac{3 - \dfrac{2}{x^2} + \dfrac{1}{x^3}}{\dfrac{2}{x} + \dfrac{1}{x^2} + \dfrac{1}{x^3}} = -\infty.$$

(a) 0 (b) $-\dfrac{3}{2}$ (c) $\dfrac{3}{2}$

(d) $-\infty$ (e) Does not exist

Name: ______________________________

Instructor: __________________________

3.(7 pts.) The slant asymptote of $y = \dfrac{2x^4 + x^3 + 5}{x^3 - 3x^2 + 2}$ is given by

Solution: Using long division, we find that

$$\frac{2x^4 + x^3 + 5}{x^3 - 3x^2 + 2} = (2x + 7) + \frac{21x^2 - 4x - 9}{x^3 - 3x^2 + 2}$$

(a) There are no slant asymptotes.

(b) $y = 2x + 7$

(c) $y = x + 4$

(d) $y = 2x - 5$

(e) $y = 2x + 4$

4.(7 pts.) Evaluate $\displaystyle\lim_{x \to -\infty} \frac{\sqrt{4x^6 + 5}}{x^3 + 1}$.

Solution: We first divide through by x^3, in the numerator using the fact that $\sqrt{x^6} = -x^3$ for $x < 0$:

$$\frac{\sqrt{4x^6 + 5}}{x^3 + 1} = -\frac{\sqrt{4 + 5/(x^6)}}{1 + 1/(x^3)}.$$

As x goes to $-\infty$, the numerator goes to $\sqrt{4} = 2$. As x goes to $-\infty$, the denominator goes to 1. Hence the answer is -2.

(a) $3/2$

(b) 6

(c) -2

(d) 2

(e) 4

Name: ______________________________

Instructor: __________________________

5.(7 pts.) If we want to use Newton's method to find an approximate solution to

$$\cos(x) - x = 0$$

with initial approximation $x_1 = \frac{\pi}{2}$, what is x_2?

Solution: Take $f(x) = \cos(x) - x$. Using that $f'(x) = -\sin(x) - 1$ and the formula $x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$, we have

$$\begin{aligned} x_2 &= \frac{\pi}{2} - \frac{-\pi/2}{-2} \\ &= \pi/2 - \pi/4 \\ &= \pi/4 \end{aligned}$$

(a) $\frac{3\pi}{4}$ (b) 0 (c) $\frac{\pi}{2}$ (d) π (e) $\frac{\pi}{4}$

6.(7 pts.) A bug being chased by a kitten (both moving in a straight line) enters a kitchen with velocity 1 ft/sec, and accelerates at $\frac{2}{\sqrt{t}}$ft/sec^2. How fast is the bug moving 9 seconds later.

Solution: Since acceleration is given by $a(t) = 2t^{-1/2}$ and the derivative of velocity is acceleration, we know that after integrating acceleration, the velocity is given by $v(t) = 4t^{1/2} + C$ for some constant C. We are also given that the initial velocity is 1 so that $v(0) = C = 1$. Thus, $v(t) = 4^{1/2} + 1$. Thus, $v(9) = 12 + 1 = 13$.

(a) 13 ft/sec (b) 7 ft/sec (c) 5 ft/sec

(d) 4 ft/sec (e) 37 ft/sec

Name: ______________________________

Instructor: __________________________

7.(7 pts.) Find the **left endpoint approximation** to the definite integral

$$\int_{-1}^{3} \frac{6}{2+x} dx$$

using four approximating rectangles of equal base width.

Solution: The left endpoint approximation uses the left endpoint of a subinterval:

$\int_a^b f(x)dx \approx \Delta x\,(f(x_0) + f(x_2) + \cdots + f(x_{n-1}))$ where $\Delta x = \dfrac{b-a}{n}$.

We have that $f(x) = \dfrac{6}{2+x}$, $a = -1$, $b = 3$, $n = 4$,

Therefore, $\Delta x = \dfrac{3-(-1)}{4} = 1$.

Divide the interval into 4 subintervals of the length $\Delta x = 1$ with the following endpoints: $-1, 0, 1, 2, 3$.

Now, just evaluate the function at the left endpoints of the subintervals.

$f(-1) = 6,\ f(0) = 3,\ f(1) = 2,\ f(2) = \dfrac{3}{2}$.

Finally, just sum up the above values and multiply Δx we obtain that

$\int_{-1}^{3} \dfrac{6}{2+x} dx \approx 1 * \left(6 + 3 + 2 + \dfrac{3}{2}\right) = \dfrac{25}{2}$.

(a) $\dfrac{71}{10}$ (b) $\dfrac{131}{10}$ (c) 25 (d) $\dfrac{71}{5}$ (e) $\dfrac{25}{2}$

8.(7 pts.) If $f(x)$ is a continuous function with

$$\int_{-2}^{-1} f(x)\,dx = 2, \quad \int_{-2}^{2} f(x)\,dx = 1 \text{ and } \int_{2}^{5} f(x)\,dx = 2$$

find $\int_{-1}^{5} f(x)\,dx$.

Soution: $\displaystyle\int_{-1}^{5} f(x)dx = \int_{-2}^{2} f(x)dx + \int_{2}^{5} f(x)dx - \int_{-2}^{-1} f(x)dx = 1 + 2 - 2 = 1.$

(a) 2 (b) 3 (c) 0 (d) 1 (e) 6

Name: ______________________________

Instructor: __________________________

9.(7 pts.) Calculate the following definite integral

$$\int_1^3 \frac{\sqrt{x}+x^3}{x^{5/2}}\,dx.$$

Solution

$$\int_1^3 \frac{\sqrt{x}+x^3}{x^{5/2}}\,dx = \int_1^3 \frac{1}{x^2}\,dx + \int_1^3 x^{1/2}\,dx = -\left(\frac{1}{3}-\frac{1}{1}\right) + \frac{2}{3}\left(3^{3/2}-1\right) = 2\sqrt{3}.$$

(a) $\frac{3}{2}$

(b) $2\sqrt{3}$

(c) $\frac{5}{2}$

(d) $2\sqrt{3}-\frac{1}{3}$

(e) $2\sqrt{3}+\frac{1}{2}$

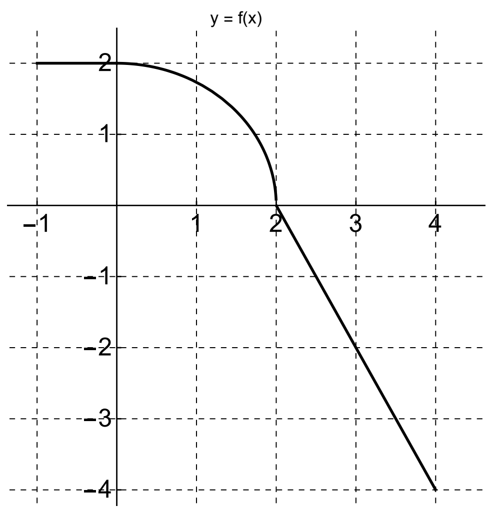
Name: ____________________________

Instructor: ________________________

10.(7 pts.) The graph shown below is that of $f(x)$ for $-1 \le x \le 4$ where

$$f(x) = \begin{cases} 2 & \text{if} -1 \le x \le 0 \\ \sqrt{4-x^2} & \text{if} 0 < x \le 2 \\ 4-2x & \text{if} 2 \le x \le 4 \end{cases}$$

Which of the following equals $\int_{-1}^{4} f(x)dx$?



Solution: From -1 to 0, the function is constant with output 2. So the area under the curve is given by the area of a rectangle with base 1 and height 2. The area of such a rectangle is 2. From 0 to 2, we see that the function traces out the upper left portion of a circle with radius 2 centered at $(0, 0)$ and hence, we have that the area is given by $\frac{1}{4}\pi 2^2 = \pi$. From 2 to 4, we see that the curve traces the hypotenuse of a right triangle with base 2 and height 4. The area of such a triangle is $\frac{1}{2}(2)(4) = 4$. However, since function is under the $x-$axis from 2 to 4, we need to account for this by subtracting the area of the triangle. So the integral is evaluated to be $2 + \pi - 4 = \pi - 2$.

(a) $\pi - 2$ (b) π (c) $6 + \pi$

(d) $2\pi - 2$ (e) 0

Name: ____________________________

Instructor: ________________________

11.(7 pts.) If $f(x) = \int_{x^3}^{1} \sqrt{1+\sin(t)}\, dt$, then $f'(x) =$

Solution: Setting $y = x^3$, we rewrite as $f(y^{1/3}) = \int_y^1 \sqrt{1+\sin(t)}\, dt$. Then we have

$$-f(y^{1/3}) = \int_1^y \sqrt{1+\sin(t)}\, dt.$$

Taking the derivative of both sides, we see via the chain rule and the Fund. Thm. of Calc. that

$$\begin{aligned} -f'(y^{1/3}) * 1/3y^{-2/3} &= \sqrt{1+\sin(y)} \\ \Rightarrow f'(y^{1/3}) &= -3y^{2/3}\sqrt{1+\sin(y)} \\ \Rightarrow f'(x) &= -3x^2\sqrt{1+\sin(x^3)} \end{aligned}$$

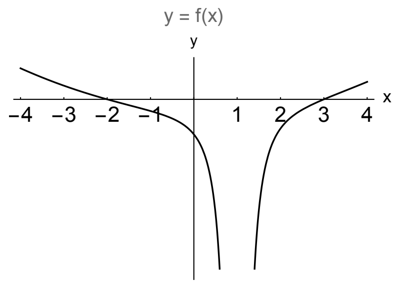
(a) $\sqrt{1+\sin(x^3)}$ (b) $\sqrt{1+\sin(x)}$ (c) $-\sqrt{1+\sin(x^3)}$

(d) $-3x^2\sqrt{1+\sin(x^3)}$ (e) $3x^2\sqrt{1+\sin(x^3)}$

Name: ______________________________

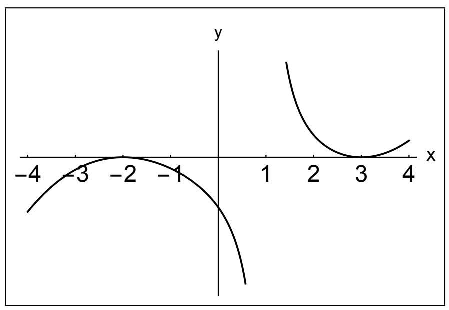
Instructor: __________________________

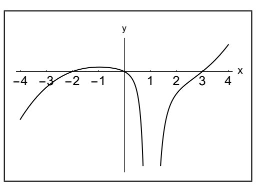
12.(7 pts.) The graph of $f(x)$ is shown below:

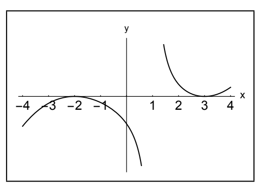


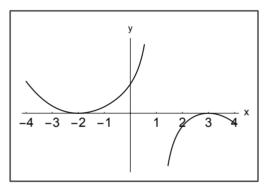
which of the following gives the graph of an antiderivative for the function $f(x)$?

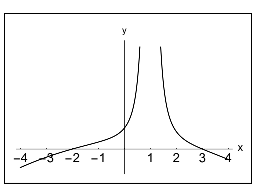
Solution Denote $F(x)$ to be one antiderivative of $f(x)$. By definition, $F'(x) = f(x)$. So $F(x)$ is increasing in $[-4, -2]$ and $[3, 4]$; decreasing in $[-2, 1) \cup (1, 3]$. The answer is

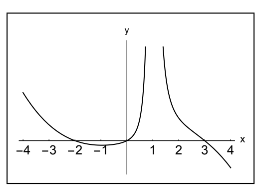


(a) 

(b) 

(c) 

(d) 

(e) 

Name: ______________________________

Instructor: __________________________

Partial Credit

You must show your work on the partial credit problems to receive credit!

13.(10 pts.) A page of a book is to have a total area of 150 square inches, with 1 inch margins at the top and sides, and a 2 inch margin at the bottom. Find the dimensions in inches of the page which will have the largest print area.

Solution Let x denote the total width and y denote the total height. So the width of the printed area is $x-2$ and the height of the printed area is $y-3$. Then the total area of the page can be expressed as

$$A_{total} = xy.$$

We are given that $A_{total} = 150$, so $y = 150/x$. We wish to maximize

$$A_{print} = (x-2)(y-3) = (x-2)\left(\frac{150}{x} - 3\right) = 156 - 3x - \frac{300}{x}.$$

Differentiating with respect to x and finding critical points gives

$$A'_{print}(x) = -3 + \frac{300}{x^2} = 0$$

so we must have $300 - 3x^2 = 0$, i.e. $x^2 = 100$. So $x = 10$ inches.

Using the first derivative test shows that 10 is indeed a maximum. For $x < 10$, $A'_{print} > 0$, and for $x > 10$, $A'_{print} < 0$.

$y = 150/x$, so we have $y = \dfrac{150}{10} = 15$. Therefore the page which maximizes the printed area has the dimensions 10 inches by 15 inches.

Name: ______________________________

Instructor: __________________________

14.(10 pts.) A particle is moving in a straight line with acceleration

$$a(t) = 4\left(t^2 - \frac{1}{3}\right) \text{ ft}/s^2,$$

where distance is measured in feet and time in seconds. The initial velocity of the particle is $v(0) = 0$ ft/s and the initial position of the particle is $s(0) = 0$.

(a) Find the velocity of the particle at time t (i.e. find $v(t)$).

Solution: $v(t) = \int a(t)\, dt = \frac{4}{3}t^3 - \frac{4}{3}t + C$. Moreover, since $v(0) = 0$, solving for C we get $C = 0$.

(b) Find the position of the particle at time t (i.e. find $s(t)$).

Solution: $s(t) = \int s(t)\, dt = \frac{1}{3}t^4 - \frac{2}{3}t^2 + C$. Moreover, since $s(0) = 0$, solving for C we get $C = 0$.

(c) Find the values of t for which $v(t) = 0$ on the interval $[0, \infty)$.

Solution: $v(t)\frac{4}{3}t(t^2 - 1) = 0$ implies $t = \pm 1$ or $t = 0$. Hence, $t = 0$ and $t = 1$ are the only values for t in $[0, \infty)$ where $v(t) = 0$.

(d) Find the <u>distance</u> travelled by the particle on the time interval $0 \leq t \leq 2$.

Solution: Observe that $v(t) < 0$ for $0 \leq t \leq 1$ and $v(t) > 0$ for $1 \leq v(t) \leq 2$. Total distance traveled is then:

$$\begin{aligned}
\int_0^2 |v(t)|\, dt &= \int_0^1 -v(t)\, dt + \int_1^2 v(t)\, dt \\
&= [-\frac{1}{3}t^4 + \frac{2}{3}t^2]_0^1 + [\frac{1}{3}t^4 - \frac{2}{3}t^2]_1^2 \\
&= -\frac{1}{3} + \frac{2}{3} + \frac{16}{3} - \frac{8}{3} - \frac{1}{3} + \frac{2}{3} = \frac{10}{3}
\end{aligned}$$

Name: ____________________________

Instructor: ________________________

15.(10 pts.) Evaluate the definite integral shown below using right endpoint approximations and the limit definition of the definite integral

$$\int_0^2 \frac{x}{2}\,dx$$

$\left(\text{Note: } 1+2+3+\cdots+n = \sum_{i=1}^{n} i = \dfrac{n(n+1)}{2}.\right)$ Verify your answer using the fundamental theorem of calculus. **Solution:** We compute $\Delta x = 2-0/n = 2/n$ and $x_i = i\Delta x = 2i/n$. Then

$$\int_0^2 \frac{x}{2}\,dx = \lim_{n\to\infty}\sum_{i=0}^{n} f(x_i)\Delta x = \lim_{n\to\infty}\sum_{i=0}^{n}\frac{2i}{2n}\left(\frac{2}{n}\right)$$

$$= \lim_{n\to\infty}\frac{2}{n^2}\sum_{i=0}^{n} i = \lim_{n\to\infty}\frac{2}{n^2}\left(\frac{n(n+1)}{2}\right) = \lim_{n\to\infty}\frac{n^2+n}{n^2} = 1.$$

Using the FTC we get

$$\int_0^2 \frac{x}{2}\,dx = \left.\frac{x^2}{4}\right|_0^2 = \frac{4}{4} - \frac{0}{4} = 1.$$

Name: ______________________________

Instructor: ANSWERS

Math 10550, Practice Exam III
November 20, 2024

- The Honor Code is in effect for this examination. All work is to be your own.
- No calculators.
- The exam lasts for 1 hour and 15 min.
- Be sure that your name is on every page in case pages become detached.
- Be sure that you have all 12 pages of the test.
- Each multiple choice question is worth 7 points. Your score will be the sum of the best 10 scores on the multiple choice questions plus your score on questions 13-15.

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1.	(a)	(b)	(•)	(d)	(e)
2.	(a)	(b)	(c)	(•)	(e)
3.	(a)	(•)	(c)	(d)	(e)
4.	(a)	(b)	(•)	(d)	(e)
5.	(a)	(b)	(c)	(d)	(•)
6.	(•)	(b)	(c)	(d)	(e)
7.	(a)	(b)	(c)	(d)	(•)
8.	(a)	(b)	(c)	(•)	(e)
9.	(a)	(•)	(c)	(d)	(e)
10.	(•)	(b)	(c)	(d)	(e)
11.	(a)	(b)	(c)	(•)	(e)
12.	(a)	(•)	(c)	(d)	(e)

Please do NOT write in this box.

Multiple Choice	______
13.	______
14.	______
15.	______
Total	______