Practice Exam 1 (2637096)

Current Score: 0/8

Question	1	2	3	4	5	6	7	8	9	10	11	12	Total
Points	0/1	0/1	0/1	0/1	0/1	0/1	0/1	0/1	**0**	**0**	**0**	**0**	**0/8**

1. 0/1 points Practice Exam 1: Q1 [1767818]

Compute

$$\lim_{x\to 1^+} \frac{1-x^2}{x^2-2x+1}.$$

- $-\infty$
- Does not exist and is not ∞ or $-\infty$.
- $+\infty$
- 1
- 0

Solution or Explanation

$$\lim_{x\to 1^+} \frac{1-x^2}{x^2-2x+1} = \lim_{x\to 1^+} \frac{(1-x)(1+x)}{(x-1)(x-1)}$$

$$= \lim_{x\to 1^+} \frac{-(1+x)}{(x-1)} = -\infty.$$

2. 0/1 points Practice Exam 1: Q2 [1767819]

For what value a is the function f given by

$$f(t) = \begin{cases} \frac{\sqrt{4+t^2}-2}{t^2} & \text{for } \ t \neq 0 \\ a & \text{for } \ t = 0 \end{cases}$$

continuous everywhere?

- No value of a makes f continuous everywhere.
- $a = \frac{1}{2}$
- any value of a
- $a = \frac{1}{4}$
- $a = 1$

Solution or Explanation

The function $f(t)$ is continuous at every point t, where $t \neq 0$, since $f(t) = \frac{\sqrt{4+t^2}-2}{t^2}$, when $t \neq 0$ and this is a quotient of continuous functions where the denominator is non-zero if $t \neq 0$.
If $f(t)$ is continuous everywhere, we must also have

$$\lim_{t \to 0} f(t) = f(0) = a.$$

That is, we must have

$$a = \lim_{t\to 0} \frac{\sqrt{4+t^2}-2}{t^2} = \lim_{t\to 0} \frac{\sqrt{4+t^2}-2}{t^2} \cdot \frac{(\sqrt{4-t^2}+2)}{(\sqrt{4+t^2}+2)}$$

$$= \lim_{t\to 0} \frac{4+t^2-4}{t^2(\sqrt{4+t^2}+2)} = \lim_{t\to 0} \frac{1}{\sqrt{4+t^2}+2} = \frac{1}{4}.$$

3. 0/1 points Practice Exam 1: Q3 [1767821]

Compute the derivative of

$$\frac{\sin x + x}{2 + \cos x}$$

.

- $\dfrac{\cos x + 1}{(2 - \sin x)^2}$
- $\dfrac{(\cos x + 1)(2 + \cos x) - (\sin x + x)\sin x}{(2 + \cos x)^2}$
- $\dfrac{\cos x + 1}{-\sin x}$
- $\dfrac{\sin x + x}{-\sin x}$
- $\dfrac{(\cos x + 1)(2 + \cos x) + (\sin x + x)\sin x}{(2 + \cos x)^2}$

Solution or Explanation

Using the quotient rule, we get

$$\begin{aligned}\frac{d}{dx}\frac{\sin x + x}{2 + \cos x} &= \frac{(2 + \cos x)\frac{d}{dx}(\sin x + x) - (\sin x + x)\frac{d}{dx}(2 + \cos x)}{(2 + \cos x)^2}\\ &= \frac{(2 + \cos x)(\cos x + 1) - (\sin x + x)(-\sin x)}{(2 + \cos x)^2}\\ &= \frac{(2 + \cos x)(\cos x + 1) + (\sin x + x)(\sin x)}{(2 + \cos x)^2}\end{aligned}$$

4. 0/1 points Practice Exam 1: Q4 [1767822]

If $f'(2) = 5$, $g(4) = 2$, $g(2) = 1$, $f(2) = -1$ and $g'(4) = 3$, find $\frac{d}{dx}(f \circ g)(4)$.

.

- 6
- 30
- -30
- -15
- 15

Solution or Explanation

$$\frac{d}{dx} f(g(x)) = f'(g(x))g'(x).$$

When $x = 4$, we get

$$\frac{d}{dx} f(g(x))\Big|_{x=4} = f'(g(4))g'(4) = f'(2)g'(4) = 5 \cdot 3 = 15$$

5. 0/1 points Practice Exam 1: Q5 [1767823]

Compute

$$\lim_{x\to 0}\frac{\sin(7x)}{\sin(5x)}.$$

- $\frac{5}{7}$
- $\frac{7}{5}$
- Does not exist.
- 1
- 0

Solution or Explanation

$$\lim_{x\to 0}\frac{\sin(7x)}{\sin(5x)} = \lim_{x\to 0}\left(\frac{\sin(7x)}{7x}\right)\left(\frac{7x}{5x}\right)\left(\frac{5x}{\sin(5x)}\right)$$

$$= \lim_{x\to 0}\left(\frac{\sin(7x)}{7x}\right)\lim_{x\to 0}\left(\frac{7x}{5x}\right)\lim_{x\to 0}\left(\frac{5x}{\sin(5x)}\right) = 1\cdot\frac{7}{5}\cdot 1 = 7/5.$$

6. 0/1 points Practice Exam 1: Q6 [1767825]

If the cost function for producing x items is given by $C(x) = 500 + 2x + 0.1x^2$ dollars, then the marginal cost at the production level of 50 unit is:

.

- 512 dollars per item
- 850 dollars per item
- 850 dollars
- 12 dollars per item.
- 22 dollars

Solution or Explanation

We need to find $C'(50)$.

$$C'(x) = 2 + 0.2x$$

therefore

$$C'(50) = 2 + 0.2(50) = 12.$$

7. 0/1 points Practice Exam 1: Q7 [1767827]

The equation of the tangent line to $y = \sin(x^2) + 1$ at $(x, y) = (\sqrt{\pi/2}, 2)$ is

- $y = (2\sqrt{\pi/2})\, x + 2$
- $y = x + 2 - \sqrt{\pi/2}$
- $x = 2$
- $y = 2$
- $y = \pi(x - \sqrt{\pi/2}) + 2.$

Solution or Explanation

Using the chain rule, we get $y' = 2x\cos(x^2)$. When $x = \sqrt{\frac{\pi}{2}}$, $y' = 2\sqrt{\frac{\pi}{2}}\cos\frac{\pi}{2} = 0$. This gives the slope of the tangent.
Using the point slope formula for a line, we find the tangent:

$$y - 2 = 0(x - \sqrt{\frac{\pi}{2}}) \quad \text{or} \quad y = 2.$$

8. 0/1 points Practice Exam 1: Q8 [1767828]

Find the derivative of $f(x) = (2 + x^4)^{2/3}$. $f'(x) =$

- $\frac{2}{3}(2 + x^4)^{-1/3}$
- $\frac{2}{3}(2 + x^4)^{-1/3}(2 + x^3)$
- $\frac{8}{3}(2 + x^4)^{-1/3}x^3$
- $\frac{2}{3}(2 + x^4)^{-1/3}x^3$
- $\frac{8}{3}(2 + x^4)^{1/3}x^3$

Solution or Explanation

Here, we use the chain rule:

$$\frac{d}{dx}(2 + x^4)^{2/3} = \frac{2}{3}(2 + x^4)^{\frac{2}{3}-1}(4x^3)$$

$$= \frac{2}{3}(2 + x^4)^{-\frac{1}{3}}(4x^3) = \frac{8}{3}(2 + x^4)^{-\frac{1}{3}}4x^3).$$

9. 0/0 points Practice Exam 1: Q9 [1767829]

Find the derivative of

$$y = \frac{1}{2x-1}$$

using the definition of the derivative.

This type of question appears as a partial credit question on the exam. Obviously you cannot submit an answer through the online homework system. Please attempt the question on paper, click on the O.K. button below and check your solution against the solution given which will appear after the due date.

.

O.K.

Solution or Explanation

Let

$$f(x) = \frac{1}{2x-1}.$$

$$\begin{aligned}
f'(x) &= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \\
&= \lim_{h\to 0} \frac{\frac{1}{2(x+h)-1} - \frac{1}{2x-1}}{h} \\
&= \lim_{h\to 0} \frac{2x+1-(2(x+h)-1)}{h(2(x+h)-1)(2x-1)} \\
&= \lim_{h\to 0} \frac{-2h}{h(2(x+h)-1)(2x-1)} \\
&= \lim_{h\to 0} \frac{-2}{(2(x+h)-1)(2x-1)} \\
&= \frac{-2}{(2x-1)^2}.
\end{aligned}$$

10. 0/0 points Practice Exam 1: Q10 [1767831]

Show that the equation

$$x^4 + 2x^3 - 2 = 0$$

has at least two solutions. Identify the theorem you use.

This type of question appears as a partial credit question on the exam. Obviously you cannot submit an answer through the online homework system. Please attempt the question on paper, click on the O.K. button below and check your solution against the solution given which will appear after the due date.

.

O.K.

Solution or Explanation

Let $f(x) = x^4 + 2x^3 - 2$. We have $f(0) = -2 < 0$ and $f(1) = 1 > 0$. Therefore, since $f(x)$ is a polynomial we can apply the Intermediate Value Theorem to show that $f(x) = 0$ has at least one solution between 0 and 1.
For large negative values of x, x^4 is positive and $|x^4| > |2x^3| \approx |2x^3 - 2|$. Therefore, we would expect $f(x) = x^4 + 2x^3 - 2$ to be positive for large negative values of x.
We use trial and error to find a negative value of x with $f(x) > 0$.

$$f(-10) = (-10)^4 + 2(-10)^3 - 2 = 10000 - 2000 - 2 > 0.$$

Therefore, by the Intermediate Value Theorem, there is at least one solution of $x^4 + 2x^3 - 2 = 0$ between -10 and 0.
This guarantees that the equation $x^4 + 2x^3 - 2 = 0$ has at least two solutions.

11. 0/0 points Practice Exam 1: Q11 [1767832]

If a ball is thrown vertically upward with a velocity of 64 ft per sec, then the distance it travelled is given by $s(t) = 64t - 16t^2$.

(a) What is the velocity after 1 second?

(b) When will the ball stop going upward? (Hint: What is the velocity when the ball stops going forward?)

(c) What is the maximum height this ball can reach?

This type of question appears as a partial credit question on the exam. Obviously you cannot submit an answer through the online homework system. Please attempt the question on paper, click on the O.K. button below and check your solution against the solution given which will appear after the due date.

.

O.K.

Solution or Explanation

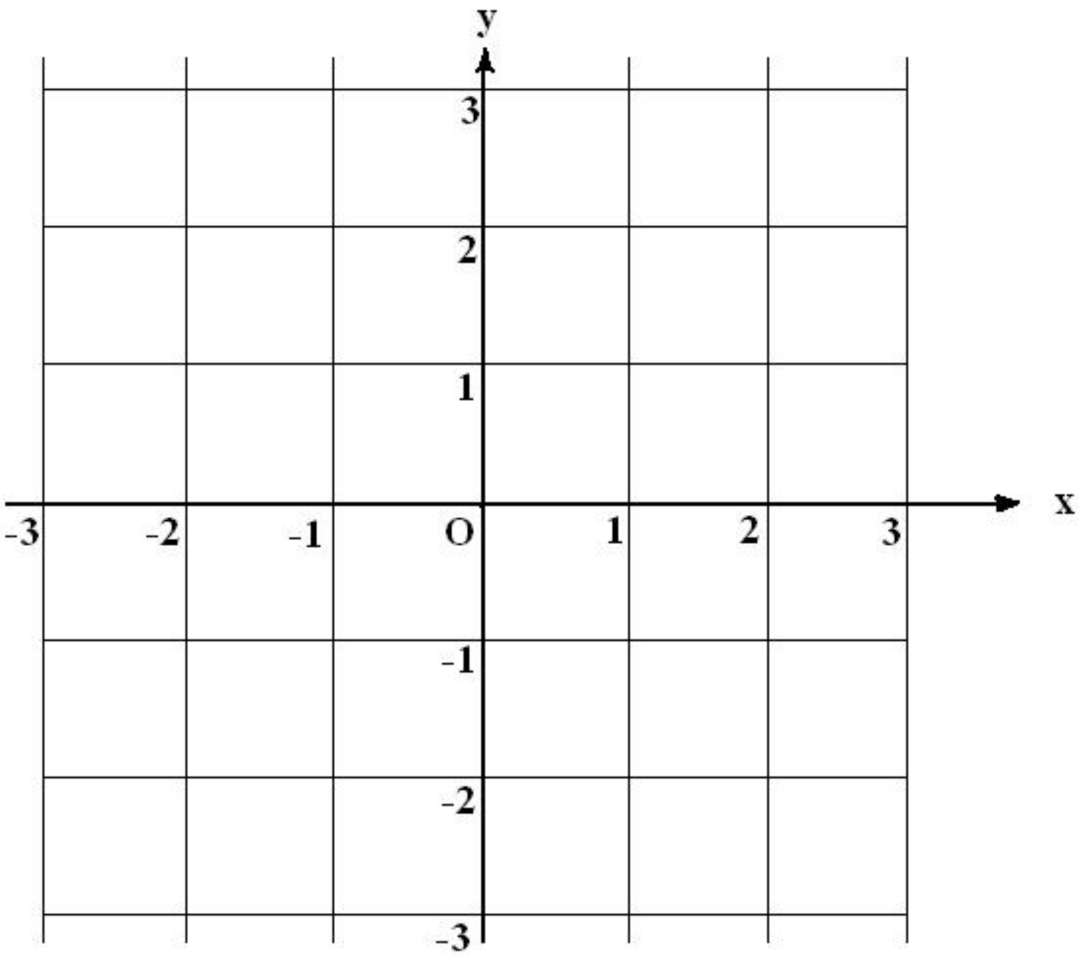
(a) The velocity is given by $v(t) = s'(t) = 64 - 32t$.
When $t = 1$, $v(1) = 64 - 32 = 32$ ft/sec.

(b) The ball turns when $v(t) = 0$.
$v(t) = 0$ if $64 - 32t = 0$, or $64 = 32t$ or $t = 2$.
Hence the ball turns after 2 seconds.

(c) The maximum height is reached when the ball turns at $t = 2$ seconds.
When $t = 2$, the height of the ball is $s(2) = 64(2) - 16(2)^2 = 128 - 64 = 64$ ft.

12. 0/0 points Practice Exam 1: Q12 [1767833]

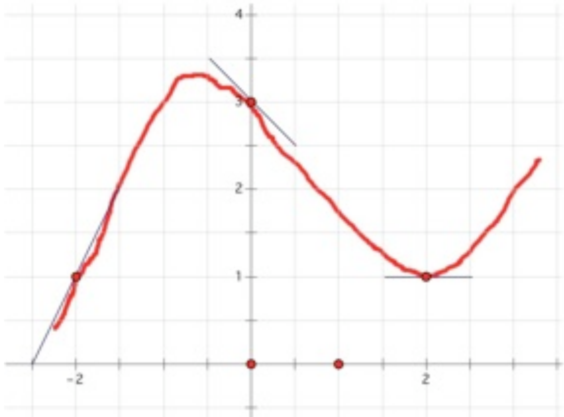
Draw the graph of a continuous function $y = f(x)$ with $f(0) = 3$, $f'(0) = -1$, $f'(2) = 0$, and $f'(-2) = 2$.



O.K.

Solution or Explanation

The following graph has the above features, but is not unique with these properties. Your answer should intercept the y-axis at $y = 3$. At that intercept the graph should be sloping downwards with a tangent line that has a slope of -1. At $x = 2$ the graph should have a horizontal tangent. At $x = -2$, the graph should have a tangent with slope 2.



Assignment Details