Math 10550, Exam III
November 27, 2007

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- No calculators.
- The exam lasts for 1 hour and 15 min.
- Be sure that your name is on every page in case pages become detached.
- Be sure that you have all 9 pages of the test.

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1. (a)  (b)  (c)  (d)  (e)
2. (a)  (b)  (c)  (d)  (e)
3. (a)  (b)  (c)  (d)  (e)
4. (a)  (b)  (c)  (d)  (e)
5. (a)  (b)  (c)  (d)  (e)
6. (a)  (b)  (c)  (d)  (e)
7. (a)  (b)  (c)  (d)  (e)
8. (a)  (b)  (c)  (d)  (e)
9. (a)  (b)  (c)  (d)  (e)
10. (a) (b)  (c)  (d)  (e)

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Multiple Choice  
11. 
12. 
13. 
Total  
Multiple Choice

1. (7 pts.) Solving the equation \( x^2 - 2 + \cos\left(\frac{\pi x}{2}\right) = 0 \) using Newton’s method with initial approximation \( x_1 = 1 \), what is \( x_2 \)?

(a) \( x_2 = \frac{1}{2} \)  
(b) \( x_2 = 1 \)  
(c) \( x_2 = \pi \)

(d) \( x_2 = \frac{\pi}{2} - 1 \)  
(e) \( x_2 = \frac{6 - \pi}{4 - \pi} \)

2. (7 pts.) The area of an ellipse

\[
\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1
\]

of semi-axis \(a\) and \(b\) is known to be \(\pi ab\). Use this (or some other geometric fact) to evaluate the integral

\[
\int_{-a}^{a} \frac{b}{a} \sqrt{a^2 - x^2} dx
\]

(a) \(\pi a^2 b^2\)  
(b) \(\pi a b^2\)  
(c) \(\sqrt{\pi} ab\)  
(d) \(2\pi a^2 b\)  
(e) \(\frac{1}{2} \pi ab\)
3. (7 pts.) Calculate the indefinite integral

\[ \int \frac{3x + 3\sqrt{x}}{\sqrt{x}} \, dx = \]

(a) \( 3x^2 + C \) \hspace{1cm} (b) \( 3x^{3/2} + C \) \hspace{1cm} (c) \( 3x + 2x^{3/2} + C \)
(d) \( x + \sqrt{x} + C \) \hspace{1cm} (e) \( x^{3/2} + C \)

4. (7 pts.) Calculate the definite integral

\[ \int_0^\pi |\cos x| \, dx = \]

(a) \( \frac{\pi}{2} \) \hspace{1cm} (b) \( \pi \) \hspace{1cm} (c) \( 1 \)
(d) \( 2 \) \hspace{1cm} (e) \( 2\pi \)
5. (7 pts.) Calculate
\[ \int 6 \tan^5 x \sec^2 x \, dx = \]
(a) \( \tan x \sec x + C \)  
(b) \( \sec^6 x + C \)  
(c) \( \tan^5 x + C \)  
(d) \( \tan^6 x + C \)  
(e) \( \sec^4 x + C \)

6. (7 pts.) Which of the following estimate holds for the integral
\[ I = \int_0^1 (1 + \cos^2 x) \, dx? \]
(a) \( 0 \leq I < \frac{\pi}{6} \)  
(b) \( 1 \leq I \leq 2 \)  
(c) \( 2 < I \leq 3 \)  
(d) \( I \leq 1 + \cos^2 1 \)  
(e) \( 0 < I < 1 \)
7. (7 pts.) Find the volume of the solid obtained by rotating the region bounded by 
\( y = x^6, \ y = 1, \) and \( x = 0, \) about the \( y \)-axis.

(a) \( 3\pi \)  \hspace{1cm} (b) \( \frac{4\pi}{3} \)  \hspace{1cm} (c) \( \frac{3\pi}{4} \)  \hspace{1cm} (d) \( 4\pi \)  \hspace{1cm} (e) \( \frac{\pi}{4} \)

8. (7 pts.) Consider the function

\[ g(x) = -\int_{\sin x}^{0} \sqrt{t^3 + 1} \, dt. \]

Then \( g'(x) = \)

(a) \( \sqrt{\sin^3 x + 1} \cos x \)  \hspace{1cm} (b) \( (\sin^3 x + 1) \cos x \)  \hspace{1cm} (c) \( 3(\sin^2 x) \sqrt{\sin^3 x + 1} \)

(d) \( \sin^3 x \cos x \)  \hspace{1cm} (e) \( \sqrt{\sin^3 x + 1} \)
9. (7 pts.) Calculate the integral
\[
\int_{-2}^{2} \frac{x^3}{1 + \cos^2 x} \, dx.
\]
(a) \(\frac{1}{16}\)  
(b) \(\frac{1}{8}\)  
(c) 0  
(d) 16  
(e) 8

10. (7 pts.) Which of the following is a Riemann sum corresponding to the integral
\[
\int_{1}^{2} \sin x \, dx?
\]
(a) \(\frac{2}{n} \sum_{i=1}^{n} \sin(1 + \frac{i}{n})\)  
(b) \(\frac{1}{n} \sum_{i=1}^{n} \sin(1 + \frac{i}{n})\)  
(c) \(\frac{1}{n} \sum_{i=1}^{n} \sin(\frac{2i}{n})\)  
(d) \(\frac{1}{n} \sum_{i=1}^{n} \sin(1 + \frac{2i}{n})\)  
(e) \(\frac{2}{n} \sum_{i=1}^{n} \sin(\frac{2i}{n})\)
11. (10 pts.) Find the area of the region bounded by the curves $y = \sin x$, $y = \cos x$ and the vertical lines $x = 0$, $x = \frac{\pi}{2}$. 
12. (10 pts.) Find the coordinates of the point on the line $x + y + 1 = 0$ that is closest to the origin. Hint: the computations are a bit easier if you minimize the square of the distance to the origin.
13. (10 pts.) A cylindrical can without a top is made to contain $\pi$ cubic centimeters of liquid. Find the dimensions (height and radius of the cylinder) that will minimize the cost of the metal to make the can.
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Multiple Choice

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13. ____________
Total ____________