Name: ______________________________

Instructor: __________________________

Math 10550, Exam III
November 19, 2013

- The Honor Code is in effect for this examination. All work is to be your own.
- No calculators.
- The exam lasts for 1 hour and 15 min.
- Be sure that your name is on every page in case pages become detached.
- Be sure that you have all 11 pages of the test.

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1.	(a)	(b)	(c)	(d)	(e)
2.	(a)	(b)	(c)	(d)	(e)
3.	(a)	(b)	(c)	(d)	(e)
4.	(a)	(b)	(c)	(d)	(e)
5.	(a)	(b)	(c)	(d)	(e)
6.	(a)	(b)	(c)	(d)	(e)
7.	(a)	(b)	(c)	(d)	(e)
8.	(a)	(b)	(c)	(d)	(e)
9.	(a)	(b)	(c)	(d)	(e)
10.	(a)	(b)	(c)	(d)	(e)

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Multiple Choice	______
11.	______
12.	______
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Total	______

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Multiple Choice

1.(6 pts.) The slant asymptote of $y = \dfrac{x^2+2x+1}{x-1}$ is given by

(a) $y = 3$ (b) $y = x + 3$ (c) $y = x$

(d) $x = 1$ (e) $y = 1$

We find the slant asymptote by long division.

$$\begin{array}{r|l} & \quad\;\; x+3 \\ \hline x-1 & x^2+2x+1 \\ & -x^2 \;\; +x \\ \hline & \qquad 3x+1 \\ & \quad -3x+3 \\ \hline & \qquad\quad 4 \end{array}$$

Thus the slant asymptote is $y = x + 3$

2.(6 pts.) The equation $x^5 + x - 1 = 0$ has one solution between 0 and 1. Find the result of one iteration of Newton's method applied to this equation with 1 as the starting point (i.e. find x_2 using Newton's method applied to the equation with $x_1 = 1$).

(a) $\dfrac{3}{4}$ (b) $\dfrac{5}{7}$ (c) $\dfrac{1}{2}$ (d) 1 (e) $\dfrac{5}{6}$

Recall that Newton's method uses the formula

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

For the given $f(x)$, we have $f(1) = 1 + 1 - 1 = 1$. Also, since $f'(x) = 5x^4 + 1$, we have $f'(1) = 5(1) + 1 = 6$. So Newton's method gives

$$x_2 = 1 - \frac{f(1)}{f'(1)} = 1 - \frac{1}{6} = \frac{5}{6}$$

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3.(6 pts.) A car racing on a straight road crosses the starting line with a velocity of 88 ft/sec. From this point on it accelerates at $\frac{60}{\sqrt{t}}$ft/sec^2. How fast in ft/sec will the car be going 4 seconds after the car has crossed the starting line?

(a) 328 ft/sec (b) 292 ft/sec (c) 208 ft/sec

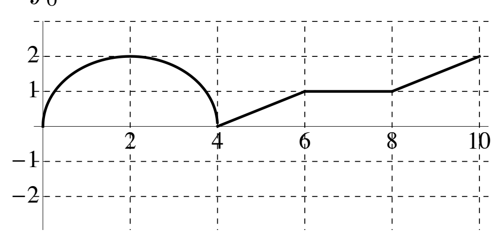
(d) 244 ft/sec (e) 152 ft/sec

Acceleration is the derivative of velocity, so to find the change in velocity we compute the definite integral of the acceleration over the time interval.

$$\int_0^4 \frac{60}{\sqrt{t}}\mathrm{d}t = 120\sqrt{t}|_0^4 = 120\sqrt{4} - 120(0) = 240$$

Thus the **change** in velocity is 240 ft/sec. We add this to the starting velocity to find that the velocity of the car after 4 seconds is $88 + 240 = 328$ ft/sec.

4.(6 pts.) The graph of a piecewise defined function $f(x)$ consisting of a semicircle and 3 straight lines, is shown below. Use the graph to calculate the value of R_5, the right endpoint approximation to $\int_0^{10} f(x)dx$ using 5 approximating rectangles.



(a) $R_5 = 8$ (b) $R_5 = 5$ (c) $R_5 = 12$

(d) $R_5 = 6$ (e) $R_5 = 16$

The function takes the values $f(2) = 2, f(4) = 0$, $f(6) = 1, f(8) = 1$, and $f(10) = 2$. The width of each rectangle is 2, since we divide up an interval 10 units long into 5 pieces.

We then get the sum $R_5 = 2f(2)+2f(4)+2f(6)+2f(8)+2f(10) = 4+0+2+2+4 = 12$

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5.(6 pts.) If $f(x) = \int_0^{5x} \cos(t^2)dt$, then $f'(x) =$

(a) $-25\cos(5x^2)$ (b) $5\cos(5x^2)$ (c) $-5\cos(5x^2)$

(d) $5\cos(25x^2)$ (e) $-5\cos(25x^2)$

Letting $g(x) = \int_0^x \cos(t^2)dt$, we see that $f(x) = g(5x)$. Then the chain rule says that $f'(x) = 5g'(5x)$. By the fundamental theorem of calculus, $g'(5x) = \cos((5x)^2)$. Thus we see that $f'(x) = 5\cos((5x)^2) = 5\cos(25x^2)$.

Alternatively: if we let $u(x) = 5x$, then $\int_0^{5x} \cos(t^2)dt = \int_0^{u} \cos(t^2)dt$ and

$$\frac{d}{dx}\int_0^u \cos(t^2)dt = \frac{d}{du}\int_0^u \cos(t^2)dt \cdot \frac{du}{dx} = [\cos(u^2)] \cdot 5 = 5\cos(25x^2).$$

6.(6 pts.) Evaluate $\int (4 - 3x^2)(4x + 1)dx$.

(a) $-36x^2 + 16 + C$ (b) $-12x^4 - 3x^3 + 16x^2 + 4x + C$

(c) $-\frac{3}{4}x^4 - x^3 + 8x^2 + 4x + C$ (d) $-2x^5 - x^4 + 8x^3 + 4x^2 + C$

(e) $-3x^4 - x^3 + 8x^2 + 4x + C$

First, we expand the integrand as $(4-3x^2)(4x+1) = -12x^3 - 3x^2 + 16x + 4$. We then can find the integral term by term, and obtain

$$\int (4 - 3x^2)(4x + 1)dx = -3x^4 - x^3 + 8x^2 + 4x + C$$

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7.(6 pts.) Evaluate the integral $\int_0^{\sqrt{\pi}} x\sin(x^2)dx$.

(a) $1 - \frac{1}{\pi}$ (b) 2 (c) 1 (d) $\frac{\pi}{4}$ (e) $\frac{1}{4}$

We perform the substitution $u = u(x) = x^2$ to simplify the integrand. Note that $\mathrm{d}u = 2x\mathrm{d}x$, $u(0) = 0, u(\sqrt{\pi}) = \pi$. Now we can write the integral as

$$\int_0^{\pi} \sin(u)\frac{\mathrm{d}u}{2}$$

Note that the upper limit has become $u(\sqrt{pi}) = \pi$ since the integral is now with respect to u. Now we find

$$\int_0^{\pi} \sin(u)\frac{\mathrm{d}u}{2} = \frac{1}{2}(-\cos(u))\Big|_0^{\pi} = \frac{1}{2}(1-(-1)) = 1$$

8.(6 pts.) Evaluate $\int_1^9 \frac{1}{\sqrt{x}(1+2\sqrt{x})^2}dx$.

(a) $\frac{1}{4}$ (b) $\frac{4}{21}$ (c) $\frac{1}{7}$ (d) 1 (e) $\frac{8}{9}$

We will use the substitution rule. Notice that if $u = u(x) = 1 + 2\sqrt{x}$ then $du = \frac{dx}{\sqrt{x}}$ and $u(1) = 3,\ u(9) = 1 + 2\sqrt{9} = 7$. Therefore, by the substitution rule

$$\int_1^9 \frac{dx}{\sqrt{x}(1+2\sqrt{x})^2} = \int_3^7 \frac{du}{(u)^2} = \frac{-1}{u}\Big|_{u=3}^{u=7} = \frac{1}{3} - \frac{1}{7} = \frac{4}{21}$$

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9.(6 pts.) Evaluate $\int_1^6 |x-2|dx$.

(a) $\frac{33}{2}$ (b) 8 (c) $\frac{17}{2}$ (d) $\frac{15}{2}$ (e) 4

First, note that

$$|x-2| = \begin{cases} x-2 & x \geq 2 \\ 2-x & x \leq 2 \end{cases}$$

and so we can write

$$\int_1^6 |x-2|dx = \int_1^2 2-x\,dx + \int_2^6 x-2\,dx = 2x - \frac{1}{2}x^2\Big|_1^2 + \frac{1}{2}x^2 - 2x\Big|_2^6$$
$$= \frac{1}{2} + 8 = \frac{17}{2}.$$

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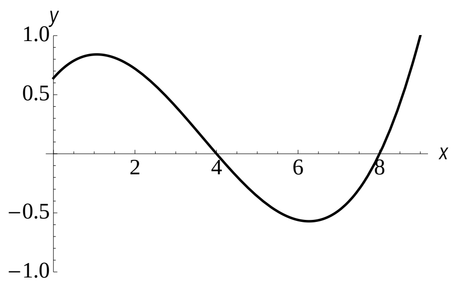
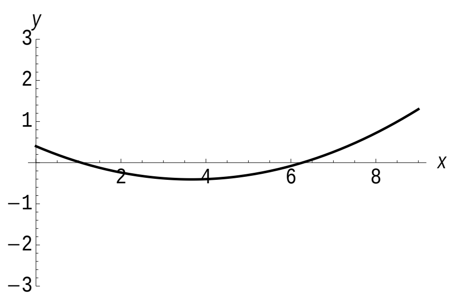
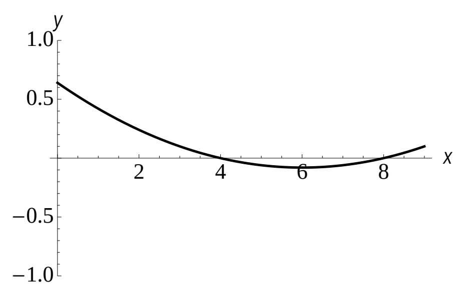
10.(6 pts.) If the following is a graph of the function $f(x)$, which graph among the answers is the graph of $\int_0^x f(t)dt$?



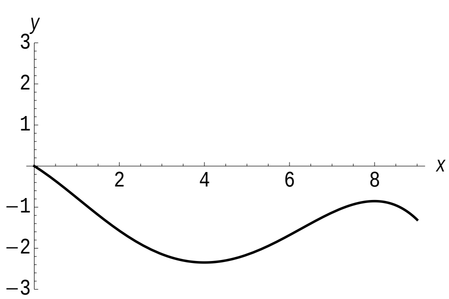
Note: The letter corresponding to the diagram is on the lower left.



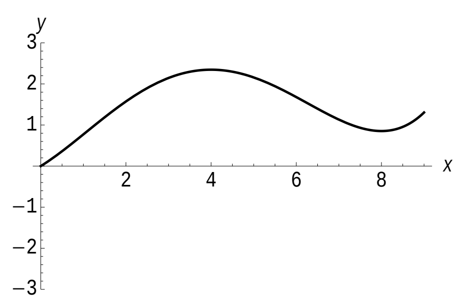
(a)



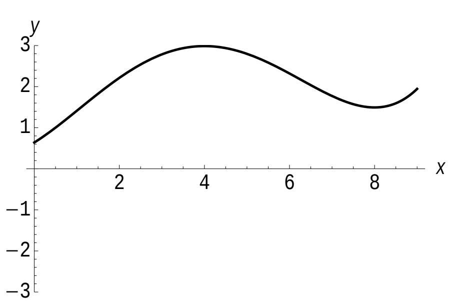
(b)



(c)



(d)



(e)

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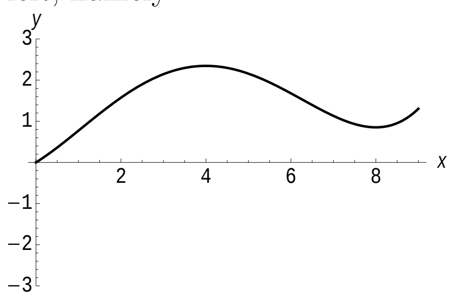
Recall that $g(x) := \int_0^x f(t)dt$ is equal to the area under the graph of $f(t)$ and that the derivative of this integral function with respect to x is $f(x)$. First notice that $g(0) = 0$, which automatically eliminates three of the choices. We see that on the interval $[0, 4]$ the function $g(x)$ is increasing, and on $[4, 6]$ the function $g(x)$ is decreasing. Thus $x = 4$ is a critical point, in fact it is a local maximum. Similarly, $g(x)$ is increasing on $[8, \infty)$, and so $x = 8$ is a local minimum. This information eliminates another choice, leaving one left, namely



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Partial Credit

You must show your work on the partial credit problems to receive credit!

11.(13 pts.) Evaluate the definite integral $\int_0^2 (1+x^2)dx$ by using right endpoint approximations and the **limit definition** of the definite integral.

Hint: $1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{1}{6}n(n+1)(2n+1)$.

When we divide up the interval $[0,2]$ into n intervals, each interval has width $\frac{2}{n}$. Thus $\Delta x = \frac{2}{n}$ and $x_i = 0 + i\Delta x = i\Delta x = \frac{2i}{n}$. The right endpoint approximation tells us that the area is approximated by $R_n = \sum_{i=1}^n f(x_i)\Delta x$.

We compute

$$\sum_{i=1}^{n} f(x_i)\Delta x = \sum_{i=1}^{n}\left(1+\frac{4i^2}{n^2}\right)\left(\frac{2}{n}\right) = \sum_{i=1}^{n}\left(\frac{2}{n}+\frac{8i^2}{n^3}\right).$$

The first term in the summation is just adding $\frac{2}{n}$ together n times, so it contributes 2 to the sum. For the second part of the sum, we use the hint.

$$\sum_{i=1}^{n}\frac{8i^2}{n^3} = \frac{8}{n^3}\sum_{i=1}^{n} i^2 = \frac{8}{n^3}\left(\frac{1}{6}n(n+1)(2n+1)\right) = \frac{4}{3}\left(\frac{n(n+1)(2n+1)}{n^3}\right)$$

(Note that in the second step we've factored out $\frac{8}{n^3}$ since it does not depend on i.)

This shows that

$$R_n = 2 + \frac{4}{3}\left(\frac{n(n+1)(2n+1)}{n^3}\right).$$

Using the limit definition of the integral, we find

$$\int_0^2 (1+x^2)dx = \lim_{n\to\infty} R_n = \lim_{n\to\infty}\left(2+\frac{4}{3}\left(\frac{n(n+1)(2n+1)}{n^3}\right)\right) = 2 + \frac{4}{3}(2) = \frac{14}{3} \text{ or } 4\frac{2}{3}.$$

Note that in going from the third to the fourth term above, we've used that

$$\lim_{n\to\infty}\frac{n(n+1)(2n+1)}{n^3} = \lim_{n\to\infty}\frac{2n^3+3n^2+n}{n^3} = 2.$$

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12.(13 pts.) Find all the points on the hyperbola $y^2 - x^2 = 4$ that are closest to the point $(2, 0)$.

The distance from a point (x, y) to the point $(2, 0)$ is given by the formula $d = \sqrt{(x-2)^2 + y^2}$. Therefore we wish to find the points (x, y) on the hyperbola for which the value

$$\sqrt{(x-2)^2 + y^2}$$

is minimal. It is equivalent to minimize the function

$$(x-2)^2 + y^2.$$

As (x, y) is on the hyperbola, we can write $y^2 = x^2 + 4$. So we must minimize

$$f(x) = (x-2)^2 + x^2 + 4$$

To minimize, we derivate with respect to x and find critical points, that is we find real zeros of

$$f'(x) = 2(x-2) + 2x = 4x - 4 = 0$$

which gives $x = 1$. Notice that $x = 1$ is indeed a minimum since for $x < 1$ one has $f'(x) < 0$ and for $x > 1$ one has $f'(x) > 0$. Notice that when $x = 1$, the only y-values for which $(1, y)$ belongs on the hyperbola are those so that $y^2 = 5$, i.e. $y = \pm\sqrt{5}$. Thus, the points on the hyperbola which are closest to $(2, 0)$ are $(1, \pm\sqrt{5})$.

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13.(14 pts.) A page of a book is to have a total area of 150 square inches, with 1 inch margins at the top and sides, and a 2 inch margin at the bottom. Find the dimensions in inches of the page which will have the largest print area.

Let x denote the total width and y denote the total height. So the width of the printed area is $x-2$ and the height of the printed area is $y-3$. Then the total area of the page can be expressed as

$$A_{total} = xy.$$

We are given that $A_{total} = 150$, so $y = 150/x$. We wish to maximize

$$A_{print} = (x-2)(y-3) = (x-2)\left(\frac{150}{x} - 3\right) = 156 - 3x - \frac{300}{x}.$$

Differentiating with respect to x and finding critical points gives

$$A'_{print}(x) = -3 + \frac{300}{x^2} = 0$$

so we must have $300 - 3x^2 = 0$, i.e. $x^2 = 100$. So $x = 10$ inches.

Using the first derivative test shows that 10 is indeed a maximum. For $x < 10$, $A'_{print} > 0$, and for $x > 10$, $A'_{print} < 0$.

$y = 150/x$, so we have $y = \dfrac{150}{10} = 15$. Therefore the page which maximizes the printed area has the dimensions 10 inches by 15 inches.

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