Multiple Choice

1. (2 pts.) A particle moving along a straight line has position function

\[ s(t) = \frac{8}{9\sqrt{t}} + \sqrt{t}, \]

where time is measured in seconds and distance is measured in feet. What is the acceleration of the particle at time \( t = 1 \) second?

Note: Recall \( t^{1/3} = \sqrt[3]{t} \).

(a) \( \frac{8}{9} \) ft/sec\(^2\)  (b) \( -\frac{1}{9} \) ft/sec\(^2\)  (c) \( \frac{2}{3} \) ft/sec\(^2\)  (d) \( \frac{4}{3} \) ft/sec\(^2\)  (e) \( \frac{4}{9} \) ft/sec\(^2\)

**Solution:** Rewrite \( s(t) \) as \( s(t) = \frac{8}{9}t^{-1/2} + t^{1/3} \). Then

\[ s'(t) = \frac{8}{9} \left( -\frac{1}{2} \right) t^{-1/2-1} + \frac{1}{3} t^{1/3-1} = -\frac{4}{9} t^{-3/2} + \frac{1}{3} t^{-2/3} \]

and

\[ s''(t) = -\frac{4}{9} \left( -\frac{3}{2} \right) t^{-3/2-1} + \frac{1}{3} \left( -\frac{2}{3} \right) t^{-2/3-1} = \frac{2}{3} t^{-5/2} - \frac{2}{9} t^{-5/3} \]

Thus the acceleration at \( t = 1 \) is \( s''(1) = \frac{2}{3} - \frac{2}{9} = \frac{4}{9} \).

2. (2 pts.) Compute

\[ \lim_{x \to 0} \frac{x + \sin(4x)}{\tan(3x)}. \]

(a) 1  (b) \(-\frac{4}{5}\)  (c) \(-\frac{2}{5}\)  (d) \(\frac{5}{3}\)  (e) \(\frac{2}{5}\)

**Solution:** Recall that \( \tan(3x) = \frac{\sin(3x)}{\cos(3x)} \). Then

\[ \lim_{x \to 0} \frac{x + \sin(4x)}{\tan(3x)} = \lim_{x \to 0} \frac{x + \sin(4x)}{\sin(3x)} \cdot \frac{\cos(3x)}{x} = \lim_{x \to 0} \frac{x + \sin(4x)}{\sin(3x)}. \]

It is clear that \( \lim_{x \to 0} \cos(3x) = 1 \).

To compute \( \lim_{x \to 0} \frac{x + \sin(4x)}{\sin(3x)} \), recall that \( \lim_{x \to 0} \frac{\sin x}{x} = 1. \)
As such, we divide both the numerator and the denominator by $x$ and get:

$$
\lim_{x \to 0} \frac{x + \sin(4x)}{\sin(3x)} = \lim_{x \to 0} \frac{1 + \frac{\sin(4x)}{x}}{\frac{\sin(3x)}{x}} = \lim_{x \to 0} \frac{1 + \frac{4 \sin(4x)}{4x}}{\frac{3 \sin(3x)}{3x}}.
$$

Since $\lim_{x \to 0} \frac{\sin(4x)}{4x} = 1$ and $\lim_{x \to 0} \frac{\sin(3x)}{3x} = 1$, it follows that

$$
\lim_{x \to 0} \frac{x + \sin(4x)}{\sin(3x)} = \frac{1 + 4}{3} = \frac{5}{3}.
$$

Hence

$$
\lim_{x \to 0} \frac{x + \sin(4x)}{\tan(3x)} = \lim_{x \to 0} \cos(3x) \cdot \lim_{x \to 0} \frac{x + \sin(4x)}{\sin(3x)} = 1 \cdot \frac{5}{3} = \frac{5}{3}.
$$
Math 10550, Quiz 4
September 26, 2017

• The Honor Code is in effect for this examination. All work is to be your own.
• No calculators.
• The exam lasts for 10 min.
• Be sure that your name is on every page in case pages become detached.
• Be sure that you have all 3 pages of the test.

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1. (a) (b) (c) (d) (●)
2. (a) (b) (c) (●) (e)