Lecture 13 : Strategy for Integration

We have the following standard table of integrals:

TABLE OF INTEGRATION FORMULAS Constants of integration have been omitted.

1. $\int x^n\,dx = \frac{x^{n+1}}{n+1} \quad (n \neq -1)$	**2.** $\int \frac{1}{x}\,dx = \ln\lvert x\rvert$
3. $\int e^x\,dx = e^x$	**4.** $\int a^x\,dx = \frac{a^x}{\ln a}$
5. $\int \sin x\,dx = -\cos x$	**6.** $\int \cos x\,dx = \sin x$
7. $\int \sec^2 x\,dx = \tan x$	**8.** $\int \csc^2 x\,dx = -\cot x$
9. $\int \sec x \tan x\,dx = \sec x$	**10.** $\int \csc x \cot x\,dx = -\csc x$
11. $\int \sec x\,dx = \ln\lvert \sec x + \tan x\rvert$	**12.** $\int \csc x\,dx = \ln\lvert \csc x - \cot x\rvert$
13. $\int \tan x\,dx = \ln\lvert \sec x\rvert$	**14.** $\int \cot x\,dx = \ln\lvert \sin x\rvert$
15. $\int \sinh x\,dx = \cosh x$	**16.** $\int \cosh x\,dx = \sinh x$
17. $\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$	**18.** $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right), \quad a > 0$
***19.** $\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert$	***20.** $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert$

Faced with an integral, we must use a problem solving approach to finding the right method or combination of methods to apply.

1. It may be possible to **Simplify the integral** e.g.

$$\int \cot x dx = \int \frac{\cos x}{\sin x} dx.$$

2. It may be possible to simplify or **solve the integral with a substitution** e.g.

$$\int \frac{1}{x(\ln x)^{10}} dx$$

3. if it is of the form

$$\int \sin^n x \cos^m x dx, \qquad \int \tan^n x \sec^m x dx \qquad \int \sin(nx)\cos(mx)dx$$

we can deal with it using the **standard methods for trigonometric functions** we have studied.

4. If we are trying to **integrate a rational function**, we apply the techniques of the previous section.

5. We should check if **integration by parts** will work.

6. If the integral contains an expression of the form $\sqrt{\pm x^2 \pm a^2}$ we can use a **trigonometric substitution.** If the integral contains an expression of the form $\sqrt[n]{ax+b}$, the function may become a rational function when we use $u = \sqrt[n]{ax+b}$, **a rationalizing substitution**. This may also work for integrals with expressions of the form $\sqrt[n]{g(x)}$ with $u = \sqrt[n]{g(x)}$

7. You may be able to **manipulate the integrand** to change its form. e.g.

$$\int \sec x dx$$

8. The integral **may resemble something you have already seen** and you may see that a change of format or substitution will convert the integral to some basic integral that you have already worked through e.g.

$$\int \sin x \cos x e^{\sin x} dx$$

9. Your solution may involve several steps.

Review

Outline how you would approach the following integrals:

1. $\int \ln x \; dx$

2. $\int \tan x \; dx$

3. $\int \sin^3 x \cos x \; dx$

4. $\int \frac{1}{\sqrt{25-x^2}} \; dx$

5. $\int \sec x \; dx$

6. $\int e^{\sqrt{x}}\, dx$

7. $\int \sin(7x)\cos(4x)\, dx$

8. $\int \cos^2 x\, dx$

9. $\int \frac{1}{x^2-9}\, dx$

More challenging integrals

The following integrals may require multiple steps:

Outline how you might approach the following integrals

$$\int \frac{x^2}{9+x^6} dx$$

$$\int \frac{1}{x^2+27x+26} dx$$

$$\int \frac{x \arctan x}{(1+x^2)^2} dx$$

$$\int \frac{\ln x}{x\sqrt{1+(\ln x)^2}}dx$$

$$\int \frac{1+\sin x}{1-\sin x}dx$$

$$\int \frac{\ln x}{\sqrt{x}}dx$$

Note There are many integrals for which our methods will not work, for example:

$$\int e^{x^2}dx, \quad \int \frac{e^x}{x}dx \quad \int \frac{1}{\ln x}dx \quad \int \frac{\sin x}{x}dx$$

see p 524 of your book for more examples. Feel free to try :)
We can estimate definite integrals of these functions using Riemann sums or the methods of the next section.