Lecture 13 : Strategy for Integration

We have the following standard table of integrals:

<table>
<thead>
<tr>
<th>TABLE OF INTEGRATION FORMULAS</th>
<th>Constants of integration have been omitted.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \int x^r dx = \frac{x^{r+1}}{r+1} ) (( r \neq -1 ))</td>
<td>2. ( \int \frac{1}{x} dx = \ln</td>
</tr>
<tr>
<td>3. ( \int e^x dx = e^x )</td>
<td>4. ( \int a^x dx = \frac{a^x}{\ln a} )</td>
</tr>
<tr>
<td>5. ( \int \sin x , dx = -\cos x )</td>
<td>6. ( \int \cos x , dx = \sin x )</td>
</tr>
<tr>
<td>7. ( \int \sec^2 x , dx = \tan x )</td>
<td>8. ( \int \csc^2 x , dx = -\cot x )</td>
</tr>
<tr>
<td>9. ( \int \sec x \tan x , dx = \sec x )</td>
<td>10. ( \int \csc x \cot x , dx = -\csc x )</td>
</tr>
<tr>
<td>11. ( \int \sec x , dx = \ln</td>
<td>\sec x + \tan x</td>
</tr>
<tr>
<td>13. ( \int \tan x , dx = \ln</td>
<td>\sec x</td>
</tr>
<tr>
<td>15. ( \int \sinh x , dx = \cosh x )</td>
<td>16. ( \int \cosh x , dx = \sinh x )</td>
</tr>
<tr>
<td>17. ( \int \frac{dx}{x^2 + a^2} = \frac{1}{a} \tan^{-1} \left( \frac{x}{a} \right) )</td>
<td>18. ( \int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1} \left( \frac{x}{a} \right), \quad a &gt; 0 )</td>
</tr>
<tr>
<td>19. ( \int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left</td>
<td>\frac{x-a}{x+a} \right</td>
</tr>
</tbody>
</table>

Faced with an integral, we must use a problem solving approach to finding the right method or combination of methods to apply.

1. It may be possible to **Simplify the integral** e.g.
   \( \int \cot x \, dx = \int \frac{\cos x}{\sin x} \, dx. \)

2. It may be possible to simplify or **solve the integral with a substitution** e.g.
   \( \int \frac{1}{x(\ln x)^3} \, dx. \)

3. If it is of the form
   \[ \int \sin^n x \cos^m x \, dx, \quad \int \tan^n x \sec^m x \, dx \quad \int \sin(nx) \cos(mx) \, dx \]
   we can deal with it using the **standard methods for trigonometric functions** we have studied.

4. If we are trying to **integrate a rational function**, we apply the techniques of the previous section.

5. We should check if **integration by parts** will work.
6. If the integral contains an expression of the form $\sqrt{\pm x^2 \pm a^2}$ we can use a trigonometric substitution. If the integral contains an expression of the form $\sqrt{ax + b}$, the function may become a rational function when we use $u = \sqrt{ax + b}$, a rationalizing substitution. This may also work for integrals with expressions of the form $\sqrt[3]{g(x)}$ with $u = \sqrt[3]{g(x)}$.

7. You may be able to manipulate the integrand to change its form. e.g.

$$\int \sec x \, dx$$

8. The integral may resemble something you have already seen and you may see that a change of format or substitution will convert the integral to some basic integral that you have already worked through e.g.

$$\int \sin x \cos x e^{\sin x} \, dx$$

9. Your solution may involve several steps.

**Review**

Outline how you would approach the following integrals:

1. $\int \ln x \, dx$

2. $\int \tan x \, dx$

3. $\int \sin^3 x \cos x \, dx$

4. $\int \frac{1}{\sqrt{25 - x^2}} \, dx$

5. $\int \sec x \, dx$
6. $\int e^{\sqrt{x}} \, dx$

7. $\int \sin(7x) \cos(4x) \, dx$

8. $\int \cos^2 x \, dx$

9. $\int \frac{1}{x^2 - 9} \, dx$

More challenging integrals
The following integrals may require multiple steps:
Outline how you might approach the following integrals

$$\int \frac{x^2}{9 + x^6} \, dx$$

$$\int \frac{1}{x^2 + 27x + 26} \, dx$$

$$\int \frac{x \arctan x}{(1 + x^2)^2} \, dx$$
\[
\int \frac{\ln x}{x \sqrt{1 + (\ln x)^2}} \, dx
\]

\[
\int \frac{1 + \sin x}{1 - \sin x} \, dx
\]

\[
\int \frac{\ln x}{\sqrt{x}} \, dx
\]

**Note** There are many integrals for which our methods will not work, for example:

\[
\int e^{x^2} \, dx, \quad \int \frac{e^x}{x} \, dx, \quad \int \frac{1}{\ln x} \, dx, \quad \int \frac{\sin x}{x} \, dx
\]

see p 524 of your book for more examples. Feel free to try :) We can estimate definite integrals of these functions using Riemann sums or the methods of the next section.