A First Order Linear Differential Equation is a first order differential equation which can be put in the form

\[ \frac{dy}{dx} + P(x)y = Q(x) \]

where \( P(x), Q(x) \) are continuous functions of \( x \) on a given interval.

The above form of the equation is called the **Standard Form** of the equation.

**Example** Put the following equation in standard form:

\[ x \frac{dy}{dx} = x^2 + 3y. \]

To solve an equation of the form

\[ \frac{dy}{dx} + P(x)y = Q(x) \]

we multiply by a function of \( x \) called an **Integrating Factor**. This function is

\[ I(x) = e^{\int P(x)dx}. \]

(we use a particular antiderivative of \( P(x) \) in this equation.)

\( I(x) \) has the property that

\[ \frac{dI(x)}{dx} = P(x)I(x). \]

Multiplying across by \( I(x) \), we get an equation of the form

\[ I(x)y' + I(x)P(x)y = I(x)Q(x). \]

The left hand side of the above equation is the derivative of the product \( I(x)y \). Therefore we can rewrite our equation as

\[ \frac{d[I(x)y]}{dx} = I(x)Q(x). \]

Integrating both sides with respect to \( x \), we get

\[ \int \frac{d[I(x)y]}{dx}dx = \int I(x)Q(x)dx \]

or

\[ I(x)y = \int I(x)Q(x)dx + C \]

giving us a solution of the form

\[ y = \frac{\int I(x)Q(x)dx + C}{I(x)} \]

(we amalgamate constants in this equation.)

**Example** Solve the differential equation

\[ x \frac{dy}{dx} = x^2 + 3y. \]
Example Solve the initial value problem

\[ y' + xy = x, \quad y(0) = -6. \]