# Euler's Method

**Extra example** The general solution to the differential equation

$$\frac{dy}{dx} = -\frac{x}{y}$$

is a family of circles centered at the origin with equations of the form $x^2 + y^2 = k^2$.

Therefore the solution to the IVP

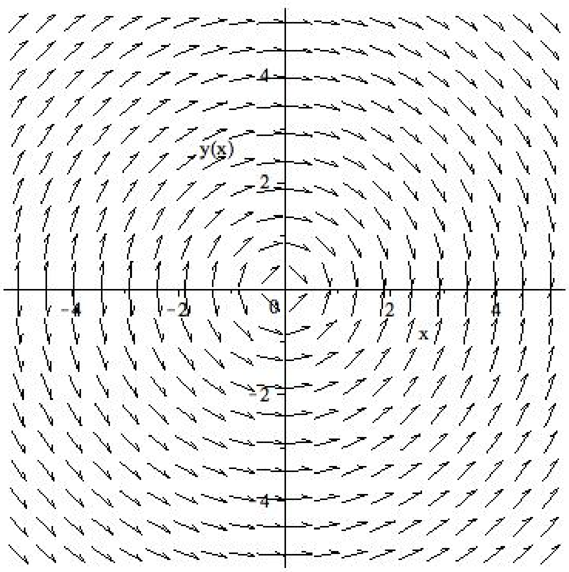
$$\frac{dy}{dx} = -\frac{x}{y}, \quad y(0) = 5$$

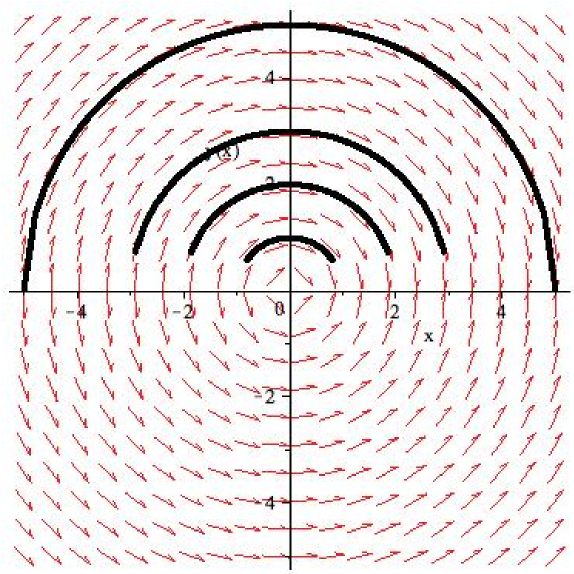
is a circle centered at the origin with radius 5 defined implicitly as $x^2 + y^2 = 25$.

Below we show a picture of the direction field for the differential equation

$$\frac{dy}{dx} = -\frac{x}{y}$$

and the graphs of some solutions.




The largest semicircle in the solutions shown in the picture on the right is the solution of our IVP given above with equation $y = \sqrt{25 - x^2}$

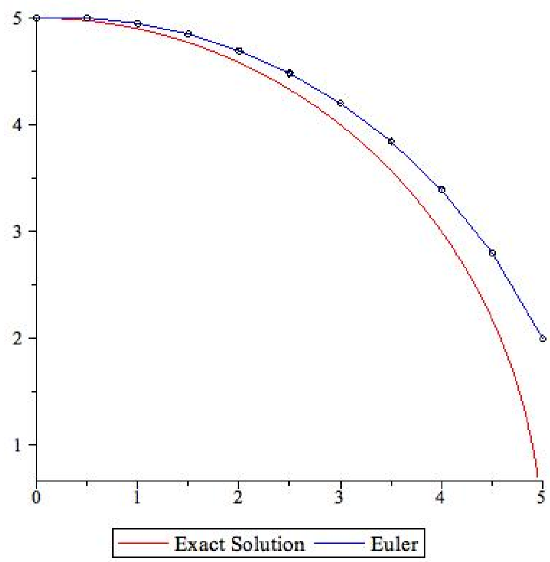
If we use Euler's method to generate a numerical solution to the IVP

$$\frac{dy}{dx} = -\frac{x}{y}, \quad y(0) = 5$$

the resulting curve should be close to this circle. The results of applying Euler's method to this initial value problem on the interval from $x = 0$ to $x = 5$ using steps of size $h = 0.5$ are shown in the table below.

| $i$ | $x_i = x_0 + ih$ | $y_i = y_{i-1} + (0.5)(\frac{-x_{i-1}}{y_{i-1}})$ |
|---|---|---|
| 0 | 0 | 5 |
| 1 | 0.5 | 5 |
| 2 | 1 | 4.95 |
| 3 | 1.5 | 4.849 |
| 4 | 2 | 4.694 |
| 5 | 2.5 | 4.481 |
| 6 | 3 | 4.202 |
| 7 | 3.5 | 3.845 |
| 8 | 4 | 3.39 |
| 9 | 4.5 | 2.8 |
| 10 | 5 | 1.997 |

A graph of the resulting approximation alongside the actual solution is shown below.



A smaller step size would result in a more accurate approximation to the curve. A graph of an approximation to the solution to the Initial value problem

$$\frac{dy}{dx} = -\frac{x}{y}, \quad y(0) = 5$$

using a step size of $h = 0.2$ is shown below. Of course this involves a lot more calculation, however it is not difficult to write a program to generate the numerical approximation using Euler's method. Many software packages have the method preprogrammed and you can find applets on the web to do the calculations. In fact in time you may be able to download (or write) an iphone app for Euler's method.