Table

TABLE OF INTEGRATION FORMULAS Constants of integration have been omitted.

1. $\int x^n\,dx = \frac{x^{n+1}}{n+1} \quad (n \neq -1)$	**2.** $\int \frac{1}{x}\,dx = \ln\lvert x\rvert$
3. $\int e^x\,dx = e^x$	**4.** $\int a^x\,dx = \frac{a^x}{\ln a}$
5. $\int \sin x\,dx = -\cos x$	**6.** $\int \cos x\,dx = \sin x$
7. $\int \sec^2 x\,dx = \tan x$	**8.** $\int \csc^2 x\,dx = -\cot x$
9. $\int \sec x \tan x\,dx = \sec x$	**10.** $\int \csc x \cot x\,dx = -\csc x$
11. $\int \sec x\,dx = \ln\lvert \sec x + \tan x\rvert$	**12.** $\int \csc x\,dx = \ln\lvert \csc x - \cot x\rvert$
13. $\int \tan x\,dx = \ln\lvert \sec x\rvert$	**14.** $\int \cot x\,dx = \ln\lvert \sin x\rvert$
15. $\int \sinh x\,dx = \cosh x$	**16.** $\int \cosh x\,dx = \sinh x$
17. $\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$	**18.** $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right), \quad a > 0$
***19.** $\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert \frac{x-a}{x+a}\right\rvert$	***20.** $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert$

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- If we are trying to **integrate a rational function**, we apply the techniques of the previous section.
- We should check if **integration by parts** will work.

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- If the integral contains an expression of the form $\sqrt{\pm x^2 \pm a^2}$ we can use a **trigonometric substitution.** If the integral contains an expression of the form $\sqrt[n]{ax+b}$, the function may become a rational function when we use $u = \sqrt[n]{ax+b}$, **a rationalizing substitution**. This may also work for integrals with expressions of the form $\sqrt[n]{g(x)}$ with $u = \sqrt[n]{g(x)}$

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$$\int \sec x dx = \int \frac{\sec x(\sec x + \tan x)}{\sec x + \tan x}$$

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- Your solution may involve several steps.

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Outline How you would approach the following integrals:

- $\int \ln x \; dx$

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- $\int \ln x \, dx$
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- $\int \frac{1}{\sqrt{25 - x^2}} \; dx$

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- Try the substitution $w = \sqrt{x}$
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- $\int e^{\sqrt{x}} \, dx = \int e^w 2w \, dw = 2 \int we^w \, dw$
- Use integration by parts now with $u = w$, $dv = e^w \, dw$.

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- $\int \sin(7x)\cos(4x)\, dx$

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- $\int \sin(7x)\cos(4x)\, dx$
- Use $\sin(mx)\cos(nx) = \frac{1}{2}\left[\sin((m-n)x) + \sin((m+n)x\right]$

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- $\int \frac{x}{x+3}\,dx$
- $\int \frac{x}{x+3}\,dx = \int \frac{u-3}{u}\,du$ where $u = x+3$.

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- $\int \frac{x^2}{9+x^6} dx$

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- $\int \frac{x \arctan x}{(1+x^2)^2} dx = \frac{-(arctanx)}{2(1+x^2)} + \int \frac{1}{2(1+x^2)^2}\ dx$

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- Partial Fractions $x^2 + 27x + 26 = (x + 26)(x + 1)$.
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- integration by parts with $u = arctan x$ and $dv = \frac{x}{(1+x^2)^2}$.
- $du = \frac{1}{1+x^2}$ and
  $v = \int \frac{x}{(1+x^2)^2}\ dx = (w = 1 + x^2) = \int \frac{1}{2} \cdot \frac{1}{w^2} = \frac{-1}{2w} = \frac{-1}{2(1+x^2)}$.
- $\int \frac{x \arctan x}{(1+x^2)^2} dx = \frac{-(arctan x)}{2(1+x^2)} + \int \frac{1}{2(1+x^2)^2}\ dx$
- For the latter integral use trig substitution $x = \tan\theta$, we get $\int sec^{-2} x\ dx = \int \cos^2 x\ dx$, we can use the half angle formula.

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- $\int \frac{\ln x}{x\sqrt{1+(\ln x)^2}} dx$

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- $= \int \sec^2 x\, dx + \int \frac{2\sin x}{\cos^2 x}\, dx + \int \tan^2 x\, dx =$
  $\tan x - \int \frac{2}{u^2}\, du + \int \sec^2 x - 1\, dx, \quad where \quad u = \cos x.$

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- $= 2\tan x + 2\sec x - x + C$

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- $= 2\tan x + 2\sec x - x + C$
- Note if you integrate this in Mathematica you get a different looking answer, but both differ by a constant

```
In[10]:= Simplify[(-x Cos[x/2] + (4 + x) Sin[x/2]) / (Cos[x/2] - Sin[x/2]) - (2 Tan[x] + 2 Sec[x] - x)]

Out[10]= -2
```

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- Let $u = \sqrt{x}$, $du = \frac{1}{2\sqrt{x}}\, dx = \frac{1}{2u}\, dx$

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The following integrals may require multiple steps:

- $\int \frac{\ln x}{\sqrt{x}} dx$
- Let $u = \sqrt{x}$, $du = \frac{1}{2\sqrt{x}}\ dx = \frac{1}{2u}\ dx$
- Let $\int \frac{\ln x}{\sqrt{x}} dx = 2 \int \ln u^2\ du$

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The following integrals may require multiple steps:

- $\int \frac{\ln x}{\sqrt{x}} dx$
- Let $u = \sqrt{x}$, $du = \frac{1}{2\sqrt{x}}\ dx = \frac{1}{2u}\ dx$
- Let $\int \frac{\ln x}{\sqrt{x}} dx = 2 \int \ln u^2\ du$
- $= 4 \int \ln u\ du$, we use integration by parts on $\int \ln u\ du$.