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A **Separable Differential Equation** is a first order differential equation of the form

$$\frac{dy}{dx} = f(y)g(x)$$

. In this case we have

$$\int \frac{1}{f(y)} dy = \int g(x) dx.$$

We can see this by differentiating both sides with respect to x. vskip .01in When we perform the above integration, we get an equation relating x and y which defines y implicitly as a function of x. Sometimes we can solve for y explicitly in terms of x.

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- $|100 - y| = e^{-x^2/2}e^{-C} \to 100 - y = \pm K e^{-x^2/2} \to y = 100 + K e^{-x^2/2}$. (can make the K negative if necessary)

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- Hence our solution is

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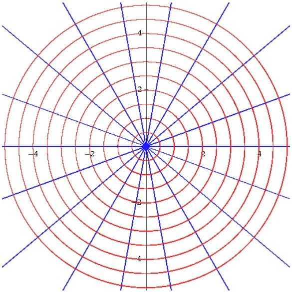
- Our solution becomes

$$y = \frac{2}{e}e^{e^x} = 2e^{(e^x - 1)}.$$

Orthogonal Trajectories.

An **Orthogonal Trajectory** of a family of curves is a curve that intersects each curve in the family of curves at right angles. Two curves intersect at right angles if their tangents at that point intersect at right angles. That is if the product of their slopes at the point of intersection is -1.

Example The family of curves $x^2 + y^2 = a^2$ is a family of concentric circles centered at the origin. The family of lines of form $y = kx$, are a family of orthogonal trajectories. Why?



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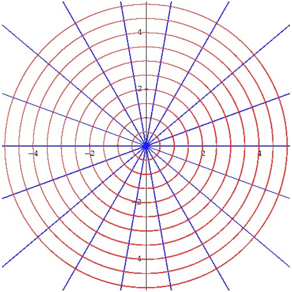

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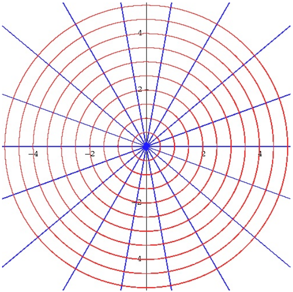

- Geometrically, we know that a tangent to a circle is perpendicular to the radial line from the point of contact to the center of the circle.
- Algebraically; At any point (x, y) on the circle $x^2 + y^2 = a^2$, we have $2x + 2y\frac{dy}{dx} = 0$. Therefore, $\frac{dy}{dx} = -\frac{x}{y}$. At the point (x, kx), we have $\frac{dy}{dx} = -\frac{1}{k}$.

Orthogonal Trajectories.

To find the orthogonal trajectories to a family of curves,

- We find a differential equation satisfied by all of the curves solving for any constants in the description in terms of x and y.
- We then use the fact that the products of the derivatives of orthogonal curves is -1 to find a new differential equation whose solution is the family of orthogonal trajectories.

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- Therefore $3\frac{y^2}{2} = -\frac{x^2}{2} + C$ or $\frac{y^2}{2/3} + \frac{x^2}{2} = C$. This is a family of ellipses.

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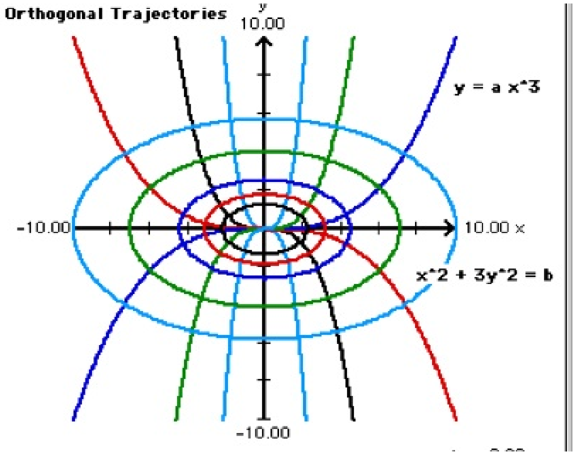

Mixture Problems.

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Rate of Change of the amount in the container	=	Rate at which chemical arrives	-	Rate at which chemical departs

Example A vat at Guinness' brewery with 500 gallons of beer contains 3% alcohol (15 gallons). Beer with 5% alcohol per unit of volume is pumped into the vat at a rate of 5 gallons/ min and the mixture is pumped out at the same rate. What is the percentage of alcohol after one hour?

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$$= \frac{25 - y}{100} \longrightarrow \boxed{\frac{dy}{dt} = \frac{25 - y}{100}.}$$

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- taking the exponential of both sides of $\ln|25 - y| = -\frac{t}{100} + C$ we get $|25 - y| = e^C e^{\frac{-t}{100}}$ or $25 - y = \pm e^C e^{\frac{-t}{100}} \quad \rightarrow \quad 25 - y = Ke^{\frac{-t}{100}} \quad \rightarrow$ $y = 25 - Ke^{\frac{-t}{100}}$.

Mixture Problems.

Example A vat at Guinness' brewery with 500 gallons of beer contains 3% alcohol (15 gallons). Beer with 5% alcohol per unit of volume is pumped into the vat at a rate of 5 gallons/ min and the mixture is pumped out at the same rate. What is the percentage of alcohol after one hour?

- Let $y(t)$ = the amount of alcohol in the container at time t. $V(t)$ = the amount of beer in the tank at time t.
- $V(t) = 500$. $y(0) = 15$ gallons.
- We have the rate of change in the amount of alcohol in the vat is

$$\boxed{\frac{dy}{dt} = \frac{25 - y}{100}.}$$

- We solve this IVP and find $y(60)$.
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- Using the initial value condition, we get $15 = 25 - Ke^0 = 25 - K$. Hence

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- $y(60) = 25 - 10e^{-0.6} = 19.512$.
- The percentage of alcohol in the tank after 60 minutes is $\frac{y(60)}{500} \times 100\% = 3.9\%$.
- Note we would expect an answer somewhere between the initial 3% and the 5% in the beer flowing into the tank.