Curves defined by Parametric equations

When the path of a particle moving in the plane is not the graph of a function, we cannot describe it using a formula that express y directly in terms of x, or x directly in terms of y. Instead, we need to use a third variable t, called a **parameter** and write:

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$$x = f(t) \qquad y = g(t)$$

- The set of points $(x, y) = (f(t), g(t))$ described by these equations when t varies in an interval I form a curve, called a **parametric curve**, and $x = f(t), y = g(t)$ are called the **parametric equations** of the curve. Often, t represents time and therefore we can think of $(x, y) = (f(t), g(t))$ as the position of a particle at time t.

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- If I is a closed interval, $a \leq t \leq b$, the point $(f(a), g(a))$ is the **initial point** and the point $(f(b), g(b))$ is the **terminal point**.

Example 1

Example 1 Draw and identify the parametric curve given by the parametric equations:

$$x = \cos t \qquad y = \sin t \qquad 0 \leq t \leq 2\pi$$

t	x	y
0		
$\frac{\pi}{4}$		
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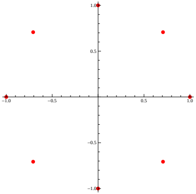
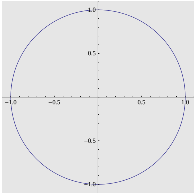
- Plotting these points, we get points on a circle.





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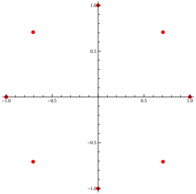

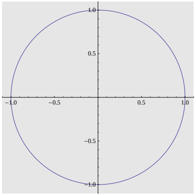

- Filling in the details, we see that if as t increases from 0 to 2π, the points trace out a circle of radius 1 in an anti-clockwise direction, with I.P. $(1, 0)$ and T.P. $(1, 0)$.

Example 2

As t increases, we travel along the curve in a particular direction giving the curve an **orientation** which is often indicated by arrows. The curve however may pass through a section of the curve repeatedly and may do so in different directions for different values of t.

Example 2 Describe the parametric curve represented by the parametric equations: $x = \sin 2t \qquad y = \cos 2t \qquad 0 \le t \le 2\pi$

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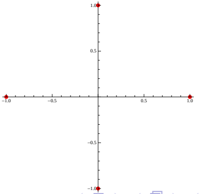
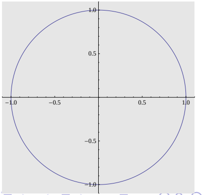
- We find some points on the curve (see table)
- We see that the points are on the unit circle, but the curve sweeps around it twice in a clockwise direction.





Example 2

Note The curve in examples 1 and 2 are the same but the parametric curve are not. Because in one case the point $(x, y) = (\cos t, \sin t)$ moves once around the circle in the counterclockwise direction starting from $(1, 0)$. In example 2 instead, the point $(x, y) = (\sin 2t, \cos 2t)$ moves twice around the circle in the clockwise direction starting from $(0, 1)$.

Example 3

Example 3 Sketch the graph of the curve described by the following set of parametric equations.

$$x = t^3 - t, \quad y = t^2, \quad 0 \leq t < \infty$$

t	x	y
0		
0.5		
1		
1.5		
2		

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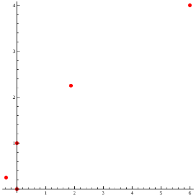
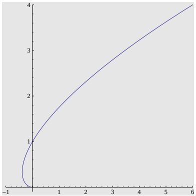
- We plot the points and join them to get a curve similar to the one shown.





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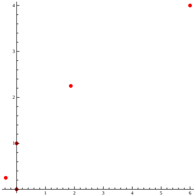

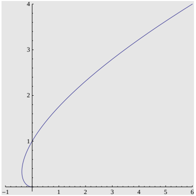

What happens after $t = 2$? Can the curve turn around again?

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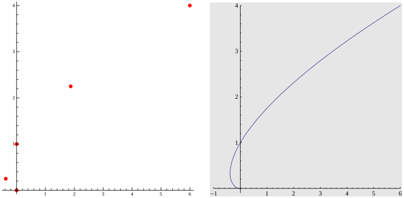

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- I claim that $x(t)$ and $y(t)$ are always increasing for $t \geq 2$ and hence the curve cannot turn again.

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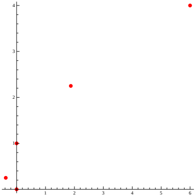

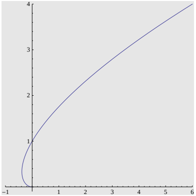

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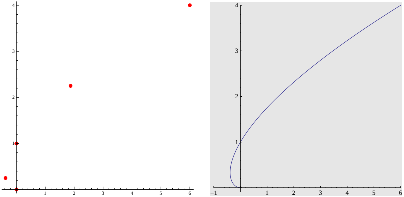

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- Since $y'(t) = 2t > 0$ for $t > 2$, we see that the values of y are increasing when $t > 2$.

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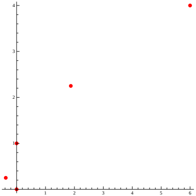
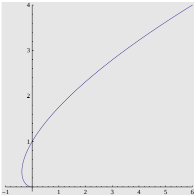

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- Since $y'(t) = 2t > 0$ for $t > 2$, we see that the values of y are increasing when $t > 2$.
- Therefore there is only one turning point on the curve.

Converting: Parametric to Cartesian

There is no exact method for converting parametric equations for a curve to an equation in x and y only. If we can solve for t in terms of either x or y, we can substitute this for the value of t in one of the equations to get an equation in x and y only.

Example 4 Covert the following parametric equation to an equation relating x and y:

$$x = 2t + 1, \quad y = t - 2, \quad -\infty < t < \infty$$

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- Substituting this expression for t in the equation for x, we get $x = 2(y + 2) + 1 = 2y + 5$.

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- Substituting this expression for t in the equation for x, we get $x = 2(y + 2) + 1 = 2y + 5$.
- Therefore all points on the parametric curve are on the line $x = 2y + 5$. Since t takes all values in the interval $(-\infty, \infty)$, y also runs through all values from $-\infty$ to ∞ and the parametric curve describes the entire line.

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Sometimes, we can see a relationship between the x and y co-ordinates and thus eliminate the t.

Example 5 Convert the following parametric equation to an equation relating x and y:

$$x = 2\cos t \qquad y = 3\sin t$$

and describe the curve traced when $0 \leq t \leq 4\pi$.

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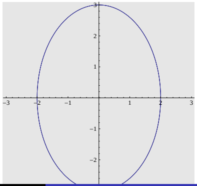
$$\left(\frac{x}{2}\right)^2 + \left(\frac{y}{3}\right)^2 = 1$$
- The curve traced by this parametric equation is therefore an ellipse. If we check some points we see that the initial point is at $(2, 0)$ and the ellipse is traced in an anti-clockwise direction. Because the point on the curve at $t + 2\pi$ is the same as the point at t, we see that the curve is traced twice.



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Easy cases If a curve is defined by the equation $y = f(x)$, the equations $x = t$ and $y = f(t)$ give parametric equations describing the curve.
If a curve is described by the equation $x = g(y)$, the equations $x = t$ and $x = g(t)$ give parametric equations describing the curve.
Example 6 Give parametric equations describing the graph of the parabola $y = x^2$.

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- We can let $x = t$, then $y = t^2$.

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If a curve is described by the equation $x = g(y)$, the equations $x = t$ and $x = g(t)$ give parametric equations describing the curve.
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- We can let $x = t$, then $y = t^2$.
- We trace out the entire parabola from left to right if we let t run from $-\infty$ to ∞.

Example: Cartesian to Parametric

Example 7 Find parametric equations on $0 \leq t \leq 2\pi$ for the motion of a particle that starts at $(a, 0)$ and traces the circle $x^2 + y^2 = a^2$ twice counterclockwise.

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- Since $0 \leq t \leq 2\pi$ implies that $0 \leq 2t \leq 4\pi$, we get that the particle sweeps around the circle twice.

Extra Example: Parametric to Cartesian

Example 5 Convert the following parametric equation to an equation relating x and y:

$$x = \sin t \qquad y = \cos^2 t$$

and describe the curve traced when $0 \leq t \leq 2\pi$.

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- We see that the particle oscillates back and forth between $(1, 0)$ and $(-1, 0)$.