Letting $dv = dx$

Example $\int_{-2}^{2} \ln(x+3)dx$.

$$\int u\, dv = uv - \int v\, du.$$

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$$\int u \, dv = uv - \int v \, du.$$

- *Let* $u = \ln(x+3)$, $dv = dx$

$$du = \frac{1}{x+3}dx \quad and \quad v = x.$$

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$$\int_{-2}^{2} \ln(x+3)dx = x\ln(x+3)\Big|_{-2}^{2} - \int_{-2}^{2} \frac{x}{x+3}\, dx$$

Letting $dv = dx$

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$$\int_{-2}^{2} \ln(x+3)dx = x\ln(x+3)\Big|_{-2}^{2} - \int_{-2}^{2} \frac{x}{x+3}\, dx$$

- *To calculate* $\int_{-2}^{2} \frac{x}{x+3}\, dx$*, we use substitution with* $w = x+3$*. Then* $x = w - 3$ *and* $w(-2) = 1$, $w(2) = 5$*. We get*

$$\int_{-2}^{2} \frac{x}{x+3}\, dx = \int_{1}^{5} \frac{w-3}{w} dw$$

$$= \int_{1}^{5} 1dw - \int_{1}^{5} \frac{3}{w} dw = 4 - 3\ln|w|\Big|_{1}^{5} = 4 - 3(\ln(5)).$$

Letting $dv = dx$

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- $$\int_{-2}^{2} \ln(x+3)dx = x\ln(x+3)\Big|_{-2}^{2} - \int_{-2}^{2} \frac{x}{x+3}\, dx$$
- $$= x\ln(x+3)\Big|_{-2}^{2} - (4 - 3\ln(5).)$$

Letting $dv = dx$

Example $\int_{-2}^{2} \ln(x+3)dx$.

$$\int u\, dv = uv - \int v\, du.$$

- $$\int_{-2}^{2} \ln(x+3)dx = x\ln(x+3)\Big|_{-2}^{2} - \int_{-2}^{2} \frac{x}{x+3}\, dx$$
- $$= x\ln(x+3)\Big|_{-2}^{2} - (4 - 3\ln(5).)$$
- $$= 2\ln(5) + 2\ln(1) - (4 - 3\ln(5).)$$
  $$= 2\ln(5) - 4 + 3(\ln(5)) = 5\ln(5) - 4.$$

Using Integration by parts twice.

Example $\int (\ln x)^2 dx$.

$$\int u\ dv = uv - \int v\ du.$$

Using Integration by parts twice.

Example $\int (\ln x)^2 dx$.

$$\int u\, dv = uv - \int v\, du.$$

- $Let\ u = (\ln(x))^2,\ dv = dx$

$$du = \frac{2\ln x}{x} dx \quad and \quad v = x.$$

Using Integration by parts twice.

Example $\int (\ln x)^2 dx$.

$$\int u\, dv = uv - \int v\, du.$$

- *Let* $u = (\ln(x))^2$, $dv = dx$

$$du = \frac{2\ln x}{x} dx \quad and \quad v = x.$$

- $$\int (\ln x)^2 dx = x(\ln x)^2 - 2\int \frac{x \ln x}{x}\, dx = x(\ln x)^2 - 2\int \ln x\, dx$$

Using Integration by parts twice.

Example $\int (\ln x)^2 dx$.

$$\int u\, dv = uv - \int v\, du.$$

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- $$\int (\ln x)^2 dx = x(\ln x)^2 - 2\int \frac{x \ln x}{x}\, dx = x(\ln x)^2 - 2\int \ln x\, dx$$
- *In a previous example, we saw that*

$$2\int \ln(x)\, dx = 2[x\ln x - x] + C$$

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- *In a previous example, we saw that*

$$2\int \ln(x)\ dx = 2[x\ln x - x] + C$$

- *Therefore*

$$\int (\ln x)^2 dx = x(\ln x)^2 - \int \ln x\ dx = x(\ln x)^2 - 2x\ln x - 2x + C$$

$$= x(\ln x)^2 - 2x\ln x + 2x + C$$

Example

Example $\int e^{2x} \cos(5x)\, dx$.

$$\int u\, dv = uv - \int v\, du.$$

Example

Example $\int e^{2x} \cos(5x)\, dx$.

$$\int u\, dv = uv - \int v\, du.$$

- *Let* $u = e^{2x}$, $dv = \cos(5x)dx$

$$du = 2e^{2x}dx \quad \textit{and} \quad v = \frac{\sin(5x)}{5}$$

Example

Example $\int e^{2x} \cos(5x)\, dx$.

$$\int u\, dv = uv - \int v\, du.$$

- *Let* $u = e^{2x}$, $dv = \cos(5x)dx$

$$du = 2e^{2x}dx \quad \text{and} \quad v = \frac{\sin(5x)}{5}$$

-

$$\int e^{2x}\cos(5x)\, dx = \frac{e^{2x}\sin(5x)}{5} - \int 2e^{2x}\frac{\sin(5x)}{5}\, dx$$

$$\int e^{2x}\cos(5x)\, dx = \frac{e^{2x}\sin(5x)}{5} - \frac{2}{5}\int e^{2x}\sin(5x)\, dx$$

Example

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$$\int u\, dv = uv - \int v\, du.$$

- *Let* $u = e^{2x}$, $dv = \cos(5x)dx$

$$du = 2e^{2x}dx \quad \textit{and} \quad v = \frac{\sin(5x)}{5}$$

- $$\int e^{2x}\cos(5x)\, dx = \frac{e^{2x}\sin(5x)}{5} - \int 2e^{2x}\frac{\sin(5x)}{5}\, dx$$
$$\int e^{2x}\cos(5x)\, dx = \frac{e^{2x}\sin(5x)}{5} - \frac{2}{5}\int e^{2x}\sin(5x)\, dx$$

- *To calculate*

$$\int e^{2x}\sin(5x)\, dx$$

we use integration by parts again.

Example

Example $\int e^{2x} \cos(5x)\, dx$.

- $$\int e^{2x} \cos(5x)\, dx = \frac{e^{2x} \sin(5x)}{5} - \frac{2}{5} \int e^{2x} \sin(5x)\, dx$$

Example

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- $$\int e^{2x}\sin(5x)\,dx$$

  Let $u = e^{2x}$ *and* $dv = \sin(5x)dx$.
  $du = 2e^{2x}$ *and* $v = -\frac{\cos(5x)}{5}$.

  $$\int e^{2x}\sin(5x)\,dx = \frac{-e^{2x}\cos(5x)}{5} + \frac{2}{5}\int e^{2x}\cos(5x)\,dx$$

Example

Example $\int e^{2x} \cos(5x)\, dx$.

- $$\int e^{2x} \cos(5x)\, dx = \frac{e^{2x}\sin(5x)}{5} - \frac{2}{5} \int e^{2x} \sin(5x)\, dx$$

- $$\int e^{2x} \sin(5x)\, dx$$

  Let $u = e^{2x}$ *and* $dv = \sin(5x)dx$.
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  $$\int e^{2x} \sin(5x)\, dx = \frac{-e^{2x}\cos(5x)}{5} + \frac{2}{5} \int e^{2x} \cos(5x)\, dx$$

- $$\int e^{2x} \cos(5x)\, dx = \frac{e^{2x}\sin(5x)}{5} - \frac{2}{5}\left[\frac{-e^{2x}\cos(5x)}{5} + \frac{2}{5} \int e^{2x} \cos(5x)\, dx\right]$$

  We now have a recurring integral.

Example

Example $\int e^{2x} \cos(5x)\, dx$.

- $$\int e^{2x} \cos(5x)\, dx = \frac{e^{2x} \sin(5x)}{5} - \frac{2}{5}\left[\frac{(-e^{2x} \cos(5x))}{5} + \frac{2}{5} \int e^{2x} \cos(5x)\, dx\right]$$

Example

Example $\int e^{2x}\cos(5x)\,dx$.

- $$\int e^{2x}\cos(5x)\,dx = \frac{e^{2x}\sin(5x)}{5} - \frac{2}{5}\left[\frac{(-e^{2x}\cos(5x))}{5} + \frac{2}{5}\int e^{2x}\cos(5x)\,dx\right]$$
- *Let* $I = \int e^{2x}\cos(5x)\,dx$

$$I = \frac{e^{2x}\sin(5x)}{5} + \frac{2}{25}e^{2x}\cos(5x) - \frac{4}{25}I.$$

Example

Example $\int e^{2x}\cos(5x)\,dx$.

- $$\int e^{2x}\cos(5x)\,dx = \frac{e^{2x}\sin(5x)}{5} - \frac{2}{5}\Big[\frac{(-e^{2x}\cos(5x))}{5} + \frac{2}{5}\int e^{2x}\cos(5x)\,dx\Big]$$
- *Let* $I = \int e^{2x}\cos(5x)\,dx$
$$I = \frac{e^{2x}\sin(5x)}{5} + \frac{2}{25}e^{2x}\cos(5x) - \frac{4}{25}I.$$
- *Let* $I = \int e^{2x}\cos(5x)\,dx$
$$(1 + \frac{4}{25})I = \frac{e^{2x}\sin(5x)}{5} + \frac{2}{25}e^{2x}\cos(5x)$$
$$\frac{29}{25}I = \frac{e^{2x}\sin(5x)}{5} + \frac{2}{25}e^{2x}\cos(5x)$$

Example

Example $\int e^{2x}\cos(5x)\,dx$.

- $$\int e^{2x}\cos(5x)\,dx = \frac{e^{2x}\sin(5x)}{5} - \frac{2}{5}\Big[\frac{(-e^{2x}\cos(5x))}{5} + \frac{2}{5}\int e^{2x}\cos(5x)\,dx\Big]$$
- *Let* $I = \int e^{2x}\cos(5x)\,dx$
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$$\frac{29}{25}I = \frac{e^{2x}\sin(5x)}{5} + \frac{2}{25}e^{2x}\cos(5x)$$
- $$I = \int e^{2x}\cos(5x)\,dx = \frac{25}{29}\left[\frac{e^{2x}\sin(5x)}{5} + \frac{2}{25}e^{2x}\cos(5x)\right].$$

Reduction formula for $\int \sin^3 x \, dx$.

Example $\int \sin^3 x dx$.

$$\int u \, dv = uv - \int v \, du.$$

Reduction formula for $\int \sin^3 x \, dx$.

Example $\int \sin^3 x dx$.

$$\int u \, dv = uv - \int v \, du.$$

- *Let* $u = \sin^2 x, \; dv = \sin x dx$

$$du = 2 \sin x \cos x dx \quad \textit{and} \quad v = -\cos x.$$

Reduction formula for $\int \sin^3 x \, dx$.

Example $\int \sin^3 x dx$.

$$\int u \, dv = uv - \int v \, du.$$

- *Let* $u = \sin^2 x, \; dv = \sin x dx$

$$du = 2 \sin x \cos x dx \quad \textit{and} \quad v = -\cos x.$$

-

$$\int \sin^3 x dx = -\cos x \sin^2 x + 2 \int (\cos x)(\sin x \cos x) \, dx$$

$$= -\cos x \sin^2 x + 2 \int \cos^2 x \sin x \, dx$$

Reduction formula for $\int \sin^3 x \, dx$.

Example $\int \sin^3 x dx$.

$$\int u \, dv = uv - \int v \, du.$$

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$$= -\cos x \sin^2 x + 2 \int \cos^2 x \sin x \, dx$$

-

$$= -\cos x \sin^2 x + 2 \int (1 - \sin^2 x) \sin x \, dx$$

$$= -\cos x \sin^2 x + 2 \int \sin x \, dx - 2 \int \sin^3 x \, dx$$

Reduction formula for $\int \sin^3 x \, dx$.

Example $\int \sin^3 x dx$.

$$\int \sin^3 x dx = -\cos x \sin^2 x + 2 \int \sin x \, dx - 2 \int \sin^3 x \, dx$$

Reduction formula for $\int \sin^3 x \, dx$.

Example $\int \sin^3 x dx$.

$$\int \sin^3 x dx = -\cos x \sin^2 x + 2 \int \sin x \, dx - 2 \int \sin^3 x \, dx$$

- *Letting* $I = \int \sin^3 x \, dx$, *we have*

$$I = -\cos x \sin^2 x + 2 \int \sin x \, dx - 2I$$

Reduction formula for $\int \sin^3 x \, dx$.

Example $\int \sin^3 x dx$.

$$\int \sin^3 x dx = -\cos x \sin^2 x + 2 \int \sin x \, dx - 2 \int \sin^3 x \, dx$$

- *Letting* $I = \int \sin^3 x \, dx$, *we have*

$$I = -\cos x \sin^2 x + 2 \int \sin x \, dx - 2I$$

- *Therefore*

$$3I = -\cos x \sin^2 x - 2\cos x + C$$

Reduction formula for $\int \sin^3 x\, dx$.

Example $\int \sin^3 x dx$.

$$\int \sin^3 x dx = -\cos x \sin^2 x + 2\int \sin x\, dx - 2\int \sin^3 x\, dx$$

- *Letting* $I = \int \sin^3 x\, dx$, *we have*

$$I = -\cos x \sin^2 x + 2\int \sin x\, dx - 2I$$

- *Therefore*

$$3I = -\cos x \sin^2 x - 2\cos x + C$$

- *and*

$$\int \sin^3 x\, dx = I = -\frac{1}{3}[\cos x \sin^2 x + 2\cos x] + C.$$