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Multiple Choice

11. 
12. 
13. 

Total
Multiple Choice

1. (6 pts) The function \( f(x) = x^3 + x + \ln(x) \) is a one-to-one function (there is no need to check this). What is \((f^{-1})'(2)\)?

(a) \( \frac{1}{5} \)  
(b) 5  
(c) \( \frac{2}{27} \)  
(d) \( \frac{27}{2} \)  
(e) \( \frac{1}{e^5} \)

2. (6 pts) Find the derivative of the function

\[
f(x) = \frac{(x^2 - 1)^5(x^2 + x + 1)^2}{\sqrt{x^2 + 1}}.
\]

(Logarithmic differentiation might help.)

(a) \( \frac{(x^2 - 1)^5(x^2 + x + 1)^2}{\sqrt{x^2 + 1}} \left[ \frac{5}{x^2 - 1} + \frac{2}{x^2 + x + 1} - \frac{1}{2(x^2 + 1)} \right] \)

(b) \( \frac{10x}{x^2 - 1} + \frac{4x + 2}{x^2 + x + 1} - \frac{x}{x^2 + 1} \)

(c) \( \frac{(x^2 - 1)^5(x^2 + x + 1)^2}{\sqrt{x^2 + 1}} \left[ \frac{5\ln(x^2 - 1)}{x^2 - 1} + \frac{2\ln(x^2 + x + 1)}{x^2 + x + 1} - \frac{\ln(x^2 + 1)}{x^2 + 1} \right] \)

(d) \( \frac{(x^2 - 1)^5(x^2 + x + 1)^2}{\sqrt{x^2 + 1}} \left[ \frac{10x}{x^2 - 1} + \frac{4x + 2}{x^2 + x + 1} - \frac{x}{x^2 + 1} \right] \)

(e) \( \frac{5\ln(x^2 - 1) + 2\ln(x^2 + x + 1)}{x^2 - 1} + \frac{\ln(x^2 + 1)}{x^2 + x + 1} \)
3. (6 pts) Compute the integral
\[ \int_0^{\log_5 3} \frac{3^x}{1 + 3^x} dx. \]
(a) \( \ln 3 (\ln 6 - \ln 2) \)  
(b) \( 3 (\ln 6 - \ln 2) \)  
(c) \( \frac{1}{\ln 3} (\ln 6 - \ln 2) \)  
(d) \( \frac{1}{3} (\ln 6 - \ln 2) \)  
(e) \( \ln 6 - \ln 2 \)

4. (6 pts) Compute the integral
\[ \int_1^e \frac{1}{x (1 + (\ln x)^2)} dx. \]
(a) \( \frac{\pi}{4} \)  
(b) \( \frac{\pi}{2} - 1 \)  
(c) \( \frac{\pi}{4} - 1 \)  
(d) \( 1 \)  
(e) \( \frac{\pi}{2} \)
5. (6 pts) Simplify the function

\[ \cos(\sin^{-1}\left(\frac{x}{2}\right)). \]

(a) \(\frac{\sqrt{4 - x^2}}{x}\)
(b) \(\frac{x}{\sqrt{4 - x^2}}\)
(c) \(\sqrt{4 - x^2}\)
(d) \(\frac{\sqrt{4 - x^2}}{2}\)
(e) \(\frac{\sqrt{4 - x^2}}{4}\)

6. (6 pts) Evaluate the limit

\[ \lim_{x \to 0^+} \frac{\tan x}{x^2}. \]

(a) 0  (b) \(-\infty\)  (c) \(-\frac{1}{2}\)  (d) \(+\infty\)  (e) \(\frac{1}{2}\)
7. (6 pts) Evaluate the integral 
\[ \int_{0}^{1} x e^{2x} \, dx. \]
(a) \( \frac{e^2 - 1}{4} \) (b) \( \frac{e^2 + 1}{2} \) (c) \( \frac{e^2 + 1}{2} \) (d) 1 (e) \( \frac{e^2 + 1}{4} \)

8. (6 pts) Evaluate the integral 
\[ \int \sin(5x) \cos(3x) \, dx. \]
Note: One of the formulas given on the last page of the exam may help you with this problem.
(a) \(-\frac{1}{2} \left[ \frac{\cos(5x)}{5} + \frac{\cos(3x)}{3} \right] + C\) (b) \(\frac{1}{2} \left[ \frac{\sin(2x)}{2} - \frac{\sin(8x)}{8} \right] + C\)
(c) \(-\frac{1}{2} \left[ \frac{\cos(2x)}{2} - \frac{\cos(8x)}{8} \right] + C\) (d) \(\frac{1}{2} \left[ \frac{\sin(2x)}{2} + \frac{\sin(8x)}{8} \right] + C\)
(e) \(-\frac{1}{2} \left[ \frac{\cos(2x)}{2} + \frac{\cos(8x)}{8} \right] + C\)
9. (6 pts) Evaluate the integral
\[ \int_{0}^{\pi/4} \tan^{100} x \sec^{4} x \, dx. \]
(a) 2  (b) \( \frac{1}{101} + \frac{1}{103} \)  (c) \( \frac{1}{100} + \frac{1}{101} \)
(d) \( \frac{1}{100} - \frac{1}{101} \)  (e) \( \frac{1}{101} - \frac{1}{103} \)

10. (6 pts) Which of the following expressions gives the correct form of the partial fraction decomposition of the function \( f \) shown below?
\[ f(x) = \frac{3x^{2} + 2x + 1}{(x - 1)(x - 4)^{2}(x^{2} + 1)^{2}} \]
(a) \( \frac{A}{x - 1} + \frac{B}{(x - 4)^{2}} + \frac{Cx + D}{(x^{2} + 1)^{2}} \)
(b) \( \frac{A}{x - 1} + \frac{B}{x - 4} + \frac{C}{(x - 4)^{2}} + \frac{D}{x^{2} + 1} + \frac{E}{(x^{2} + 1)^{2}} \)
(c) \( \frac{A}{x - 1} + \frac{B}{x - 4} + \frac{C}{(x - 4)^{2}} + \frac{Dx + E}{x^{2} + 1} + \frac{Fx + G}{(x^{2} + 1)^{2}} \)
(d) \( \frac{A}{x - 1} + \frac{Bx + C}{(x - 4)^{2}} + \frac{Dx + E}{(x^{2} + 1)^{2}} \)
(e) \( \frac{A}{x - 1} + \frac{B}{(x - 4)^{2}} + \frac{C}{(x^{2} + 1)^{2}} \)
11. (10 pts.) let $M(t)$ denote the amount of a chemical substance remaining after $t$ years where the initial amount is given by $M(0)$. The rate of decay of the substance is such that 40% of the initial amount is left after 10 years. It is known that the substance decreases at a rate proportional to the amount present a time $t$, that is $C'(t) = kC(t)$ for some constant $k$.

(a) What is the value of $k$?

(b) What is the half-life of this substance (what is the amount of time it takes to decay to 50% of its original size)?
12. (15 pts.) Compute the integral
\[ \int \frac{x^2 + 3x}{(x - 2)(x^2 + 2x + 2)} \, dx. \]
13. (15 pts.) Calculate the integral
\[ \int_0^1 \frac{x^2}{\sqrt{4 - x^2}} \, dx . \]
Note: One of the formulas given on the last page of the exam may help you with this problem.
The following is the list of useful trigonometric formulas:

\[
\sin^2 x + \cos^2 x = 1
\]

\[
1 + \tan^2 x = \sec^2 x
\]

\[
\sin^2 x = \frac{1}{2}(1 - \cos 2x)
\]

\[
\cos^2 x = \frac{1}{2}(1 + \cos 2x)
\]

\[
\sin 2x = 2 \sin x \cos x
\]

\[
\sin x \cos y = \frac{1}{2} ( \sin(x - y) + \sin(x + y) )
\]

\[
\sin x \sin y = \frac{1}{2} ( \cos(x - y) - \cos(x + y) )
\]

\[
\cos x \cos y = \frac{1}{2} ( \cos(x - y) + \cos(x + y) )
\]

\[
\int \sec \theta = \ln | \sec \theta + \tan \theta | + C
\]
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