Name: ___________________________

Instructor: ________________________

Math 10560, Exam 2
March 20, 2014

- The Honor Code is in effect for this examination. All work is to be your own.
- No calculators.
- The exam lasts for 1 hour and 15 min.
- Be sure that your name is on every page in case pages become detached.
- Be sure that you have all 11 pages of the test.

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1.	(a)	(b)	(c)	(d)	(e)
2.	(a)	(b)	(c)	(d)	(e)
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9.	(a)	(b)	(c)	(d)	(e)
10.	(a)	(b)	(c)	(d)	(e)

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Multiple Choice	________
11.	________
12.	________
13.	________
14.	________
Total	________

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Multiple Choice

1.(6 pts) Use Simpson's rule with $n = 4$ to estimate

$$\tan^{-1}(2) = \int_0^2 \frac{1}{1+x^2}dx.$$

(a) $\frac{1}{6}\left[1 + \frac{16}{5} + 1 + \frac{16}{13} + \frac{1}{5}\right]$

(b) $\frac{1}{6}\left[1 + \frac{4}{5} + \frac{1}{2} + \frac{4}{13} + \frac{1}{5}\right]$

(c) $\frac{1}{4}\left[1 + \frac{16}{5} + 1 + \frac{16}{13} + \frac{1}{5}\right]$

(d) $\frac{1}{4}\left[1 + \frac{8}{5} + 1 + \frac{8}{13} + \frac{1}{5}\right]$

(e) $\frac{1}{6}\left[1 + \frac{8}{5} + 1 + \frac{8}{13} + \frac{1}{5}\right]$

2.(6 pts) Evaluate the improper integral

$$\int_3^5 \frac{1}{\sqrt{x-3}}dx.$$

(a) The integral diverges

(b) 0

(c) $\frac{4\sqrt{2}}{3}$

(d) $2\sqrt{2}$

(e) $-\frac{4\sqrt{2}}{3}$

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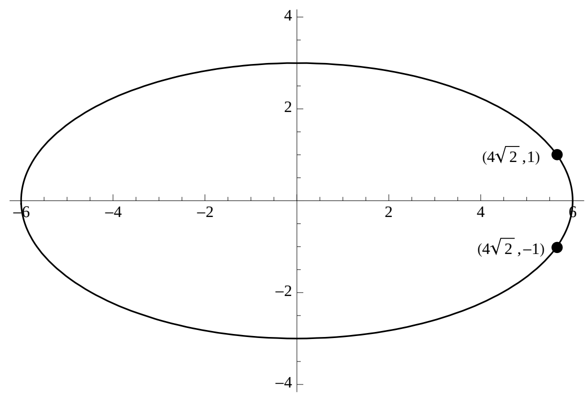
3.(6 pts)

Which of the following integrals corresponds to the length of the shorter arc of the ellipse

$$\frac{x^2}{4} + y^2 = 9$$

(shown in the picture at right) from the point $(4\sqrt{2}, -1)$ to the point $(4\sqrt{2}, 1)$.



(a) $\displaystyle\int_{-1}^{1} \Big(\frac{9+3y^2}{9-y^2}\Big) dy.$

(b) $\displaystyle\int_{-1}^{1} \sqrt{37-4y^2}\, dy.$

(c) $\displaystyle\int_{-4\sqrt{2}}^{4\sqrt{2}} \sqrt{\frac{9+3y^2}{9-y^2}}\, dy.$

(d) $\displaystyle\int_{-4\sqrt{2}}^{4\sqrt{2}} \sqrt{1-\frac{2y}{\sqrt{9-y^2}}}\, dy.$

(e) $\displaystyle\int_{-1}^{1} \sqrt{\frac{9+3y^2}{9-y^2}}\, dy.$

4.(6 pts) Evaluate the improper integral

$$\int_1^{\infty} \frac{x}{e^{x^2/2}} dx.$$

(a) $\dfrac{1}{\sqrt{e}}$ (b) $\sqrt{e}$ (c) $\dfrac{1}{e}$

(d) The integral diverges (e) 1

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5.(6 pts) Use Euler's method with step size 0.2 to estimate $y(0.4)$ where $y(x)$ is the solution to the initial value problem

$$y' = 10(x+y)^2, \quad y(0) = 0.$$

(a) 0.8 (b) 0.08 (c) 0 (d) 2.8 (e) 0.4

6.(6 pts) Find the solution of the differential equation:

$$\frac{dy}{dx} = \frac{x+1}{e^y},$$

with initial condition $y(0) = 2$.

(a) $y = \ln\left|\frac{x^2}{2} + x + 1\right| + 2$

(b) $y = 2e^{([x^2/2]+x)}$

(c) $y = 2 + e^{([x^2/2]+x)}$

(d) $y = \ln\left|\frac{x^2}{2} + x + e^2\right|$

(e) $y = 2 + \ln|\ln|\frac{x^2}{2} + x + 1||$

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7.(6 pts) Find the general solution of the differential equation:

$$y' - \left(\frac{1}{x}\right)y = 1 + x^2.$$

(a) $x\Big(\ln|x| + \dfrac{x^2}{2}\Big) + C$

(b) $x\Big(\ln|x| + \dfrac{x^2}{2} + C\Big)$

(c) $\dfrac{x}{2} + \dfrac{x^2}{3} + \dfrac{C}{x}$

(d) $x + \dfrac{x}{\ln x} + \dfrac{C}{\ln x}$

(e) $\dfrac{x}{2} + \dfrac{x^2}{3} + C$

8.(6 pts) Determine if the sequence given by $a_n = \dfrac{\tan^{-1}(n)}{n}$ converges or diverges, and if it converges find

$$\lim_{n\to\infty} \frac{\tan^{-1}(n)}{n}.$$

(a) The sequence converges and $\lim_{n\to\infty} a_n = 2$.

(b) The sequence diverges.

(c) The sequence converges and $\lim_{n\to\infty} a_n = 0$.

(d) The sequence onverges and $\lim_{n\to\infty} a_n = \dfrac{\pi}{2}$.

(e) The sequence converges and $\lim_{n\to\infty} a_n = \dfrac{2}{\pi}$.

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9.(6 pts) Consider the following sequences:

$$(I)\ \left\{(-1)^n\frac{n^2-1}{2^n}\right\}_{n=1}^{\infty} \qquad (II)\ \left\{(-1)^n\frac{n^2-1}{2n^2}\right\}_{n=1}^{\infty} \qquad (III)\ \left\{(-1)^n n\ln(n)\right\}_{n=1}^{\infty}$$

Which of the following statements is true?

(a) Sequence I converges but sequences II and III diverge.

(b) Sequences I and II converge but sequence III diverges.

(c) Sequences II and III converge but sequence I diverges.

(d) All three sequences diverge.

(e) All three sequences converge.

10.(6 pts) Find the sum of the following series:

$$\sum_{n=1}^{\infty}\frac{(-1)^n 2^{n+1}}{3^n}.$$

(a) $-\frac{3}{5}$

(b) $\frac{3}{5}$

(c) This series diverges.

(d) $\frac{4}{5}$

(e) $-\frac{4}{5}$

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Partial Credit

You must show your work on the partial credit problems to receive credit!

11. (13 pts.) Find the family of orthogonal trajectories to the family of curves given by

$$y = k(\sqrt[3]{x}).$$

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12. (13 pts.) Find the arc length of the curve $y = f(x)$ from the point $(0, 1/3)$ to the point $(1, \frac{e^3 + e^{-3}}{6})$, where

$$f(x) = \frac{e^{3x} + e^{-3x}}{6}.$$

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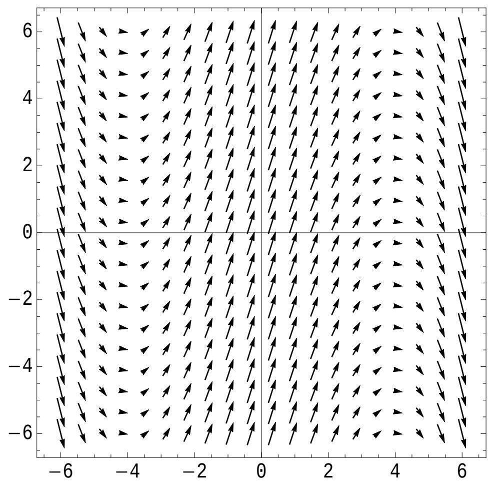
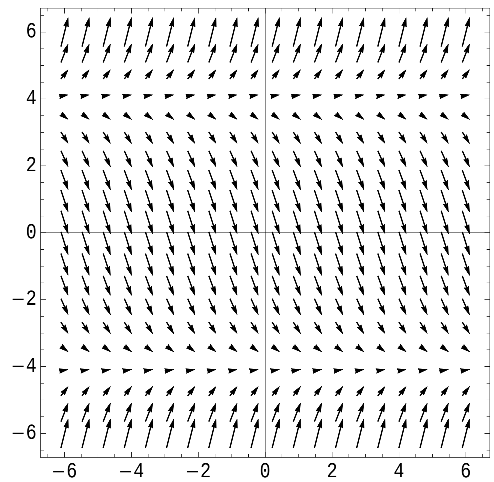
13. (12 pts.) (a) Which of the pictures below show the direction field for the differential equation

$$\frac{dy}{dx} = (4 - y)(4 + y).$$

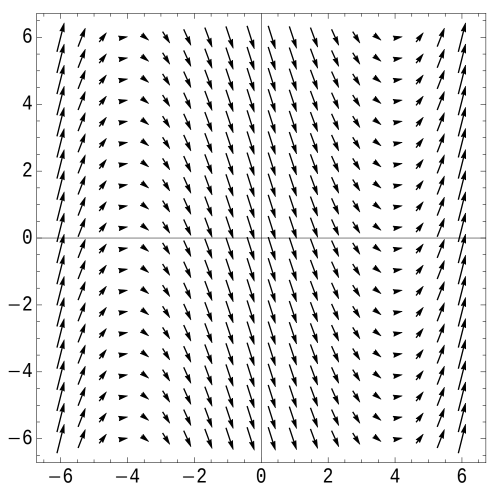
Circle the label at the lower left of your answer to indicate your choice. Justify your answer with some calculations; enough to distinguish your choice from the other options. Note that the point $(0, 0)$ is in the center of each picture.



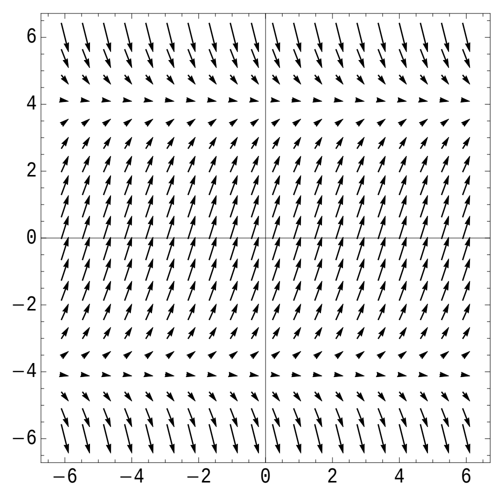
(I)



(II)



(III)



(IV)

(b) On the direction field you have selected above , sketch the graph of the solution with initial condition $y(0) = \frac{3}{2}$.

(c) For the solution you have sketched in part (b), use the direction field to determine $\lim_{x\to\infty} y(x)$?

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14. (2 pts.) You get one of these two points for marking your answers on the front page with X's (not circles) and you get the second point if your instructor can read your name at a glance.
Also you can use this page for rough work.

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The following is the list of useful trigonometric formulas:

$$\sin^2 x + \cos^2 x = 1$$

$$1 + \tan^2 x = \sec^2 x$$

$$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$$

$$\cos^2 x = \frac{1}{2}(1 + \cos 2x)$$

$$\sin 2x = 2 \sin x \cos x$$

$$\sin x \cos y = \frac{1}{2}\big(\sin(x - y) + \sin(x + y)\big)$$

$$\sin x \sin y = \frac{1}{2}\big(\cos(x - y) - \cos(x + y)\big)$$

$$\cos x \cos y = \frac{1}{2}\big(\cos(x - y) + \cos(x + y)\big)$$

$$\int \sec\theta = \ln|\sec\theta + \tan\theta| + C$$

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