Be sure that you have all 14 pages of the test.
No calculators are to be used.
The exam lasts for two hours.
When told to begin, remove this answer sheet and keep it under the rest of your test. When told to stop, hand in just this one page.
The Honor Code is in effect for this examination, including keeping your answer sheet under cover.

<p>| | | | | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(a)</td>
<td>(b)</td>
<td>(c)</td>
<td>(d)</td>
<td>(e)</td>
<td>15</td>
<td>(a)</td>
<td>(b)</td>
<td>(c)</td>
</tr>
<tr>
<td>2</td>
<td>(a)</td>
<td>(b)</td>
<td>(c)</td>
<td>(d)</td>
<td>(e)</td>
<td>16</td>
<td>(a)</td>
<td>(b)</td>
<td>(c)</td>
</tr>
<tr>
<td>3</td>
<td>(a)</td>
<td>(b)</td>
<td>(c)</td>
<td>(d)</td>
<td>(e)</td>
<td>17</td>
<td>(a)</td>
<td>(b)</td>
<td>(c)</td>
</tr>
<tr>
<td>4</td>
<td>(a)</td>
<td>(b)</td>
<td>(c)</td>
<td>(d)</td>
<td>(e)</td>
<td>18</td>
<td>(a)</td>
<td>(b)</td>
<td>(c)</td>
</tr>
<tr>
<td>5</td>
<td>(a)</td>
<td>(b)</td>
<td>(c)</td>
<td>(d)</td>
<td>(e)</td>
<td>19</td>
<td>(a)</td>
<td>(b)</td>
<td>(c)</td>
</tr>
<tr>
<td>6</td>
<td>(a)</td>
<td>(b)</td>
<td>(c)</td>
<td>(d)</td>
<td>(e)</td>
<td>20</td>
<td>(a)</td>
<td>(b)</td>
<td>(c)</td>
</tr>
<tr>
<td>7</td>
<td>(a)</td>
<td>(b)</td>
<td>(c)</td>
<td>(d)</td>
<td>(e)</td>
<td>21</td>
<td>(a)</td>
<td>(b)</td>
<td>(c)</td>
</tr>
<tr>
<td>8</td>
<td>(a)</td>
<td>(b)</td>
<td>(c)</td>
<td>(d)</td>
<td>(e)</td>
<td>22</td>
<td>(a)</td>
<td>(b)</td>
<td>(c)</td>
</tr>
<tr>
<td>9</td>
<td>(a)</td>
<td>(b)</td>
<td>(c)</td>
<td>(d)</td>
<td>(e)</td>
<td>23</td>
<td>(a)</td>
<td>(b)</td>
<td>(c)</td>
</tr>
<tr>
<td>10</td>
<td>(a)</td>
<td>(b)</td>
<td>(c)</td>
<td>(d)</td>
<td>(e)</td>
<td>24</td>
<td>(a)</td>
<td>(b)</td>
<td>(c)</td>
</tr>
<tr>
<td>11</td>
<td>(a)</td>
<td>(b)</td>
<td>(c)</td>
<td>(d)</td>
<td>(e)</td>
<td>25</td>
<td>(a)</td>
<td>(b)</td>
<td>(c)</td>
</tr>
<tr>
<td>12</td>
<td>(a)</td>
<td>(b)</td>
<td>(c)</td>
<td>(d)</td>
<td>(e)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>(a)</td>
<td>(b)</td>
<td>(c)</td>
<td>(d)</td>
<td>(e)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>(a)</td>
<td>(b)</td>
<td>(c)</td>
<td>(d)</td>
<td>(e)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
1. (6 pts.) The function \( f(x) = 2x + \ln x \) is one-to-one. Compute \((f^{-1})'(2)\).

(a) \( \frac{1}{3} \)  
(b) \( \frac{5}{2} \)  
(c) \( 4 + \ln 2 \)

(d) \( \frac{2}{5} \)  
(e) 0

2. (6 pts.) Solve the equation \( \log_4(x) + \log_4(x^2) = -\frac{3}{2} \). Then \( x = \)

(a) \( \frac{1}{\sqrt{e}} \)  
(b) 2  
(c) \( \frac{1}{2} \)  
(d) \( \frac{3}{2} \)  
(e) \( -2 \)
3. (6 pts.) Use logarithmic differentiation to compute the derivative of the function

\[ f(x) = \frac{2^x (x^3 + 1)}{\sqrt{x + 1}}. \]

(a) \[ f'(x) = \frac{2^x (x^3 + 1)}{\sqrt{x + 1}} \left( \ln 2 + \frac{3x^2}{x^3 + 1} - \frac{1}{2(x + 1)} \right) \]

(b) \[ f'(x) = \frac{2^x (x^3 + 1)}{\sqrt{x + 1}} \left( \frac{1}{2} + \frac{3x^2}{x^3 + 1} - \frac{1}{2(x + 1)} \right) \]

(c) \[ f'(x) = \frac{2^x (x^3 + 1)}{\sqrt{x + 1}} \left( \frac{1}{\ln 2} + \frac{1}{x^3 + 1} - \frac{1}{x + 1} \right) \]

(d) \[ f'(x) = \frac{2^x (x^3 + 1)}{\sqrt{x + 1}} \left( 2 + \frac{1}{x^3 + 1} - \frac{1}{x + 1} \right) \]

(e) \[ f'(x) = \frac{2^x (x^3 + 1)}{\sqrt{x + 1}} \left( \frac{1}{\ln 2} + \frac{3x^2}{x^3 + 1} - \frac{1}{2(x + 1)} \right) \]

4. (6 pts.) You begin an experiment at 9am with a sample of 1000 bacteria. An hour later your population has doubled. Assuming exponential growth, what is the population at noon?

(a) 32,000  (b) 4,000  (c) 8,000  (d) $1,000e^{-3}$  (e) $1,000e^3$
5. (6 pts.) Simplify $\sin^{-1}(\sin \frac{9\pi}{10})$.

(a) $-\frac{\pi}{10}$
(b) $\frac{9\pi}{10}$
(c) not enough information to tell.
(d) $\frac{\pi}{10}$
(e) 0

6. (6 pts.) Compute the limit $\lim_{x \to \infty} (2x)^{\frac{1}{2}}$.

(a) 1 (b) 0 (c) $e$ (d) 2 (e) $\infty$
7. (6 pts.) Evaluate the integral
\[ \int_{0}^{\pi} x \sin(x) \, dx. \]
(a) 0  (b) 1  (c) −1  (d) \pi  (e) \frac{\pi}{2}

8. (6 pts.) Find the integral \( \int_{0}^{2} \sqrt{4 - x^2} \, dx. \)
(a) 0  (b) 4 + \sin 4  (c) \pi  (d) \pi + 2  (e) −2
9. (6 pts.) Evaluate the integral
\[ \int \frac{x - 9}{x^2 + 3x - 10} \, dx. \]

(a) \( \ln \left| \frac{x - 5}{x + 2} \right| + C \)  
(b) \( \ln \left| \frac{x - 2}{x + 5} \right| + C \)  
(c) \( \ln \left| \frac{(x + 2)^2}{(x - 5)^3} \right| + C \)

(d) \( \ln \left| \frac{(x + 5)^2}{x - 2} \right| + C \)  
(e) \( \ln \left| \frac{x + 5}{(x - 2)^2} \right| + C \)

10. (6 pts.) Determine whether the following integral converges or diverges. If it converges, evaluate.
\[ \int_{-2}^{0} \frac{1}{(x + 1)^2} \, dx. \]

(a) Converges to -2.  (b) Converges to 0.  (c) Converges to 2.
(d) Diverges.  (e) Converges to 1.
11. (6 pts.) Find the length of the curve

\[ y = 1 + \frac{4}{3}x^{3/2} \quad \text{for} \ 0 \leq x \leq 1. \]

(a) \( \frac{1}{6}(1 - \sqrt{5}) \)  
(b) \( \frac{2}{3}(1 - \sqrt{5}) \)  
(c) \( \frac{1}{12}(3\sqrt{3} - 1) \)  
(d) \( \frac{1}{12}(11\sqrt{5} - 1) \)  
(e) \( \frac{1}{6}(5\sqrt{5} - 1) \)

12. (6 pts.) Find the centroid of the region bounded by \( y = x^2 \) and \( y = x \).

(a) \( \left( \frac{1}{2}, \frac{1}{2} \right) \)  
(b) \( \left( \frac{1}{10}, \frac{2}{5} \right) \)  
(c) \( \left( \frac{1}{10}, \frac{1}{15} \right) \)  
(d) \( \left( \frac{1}{12}, \frac{1}{15} \right) \)  
(e) \( \left( \frac{1}{2}, \frac{2}{5} \right) \)
13. (6 pts.) The solution to the initial value problem
\[ y' = \frac{\sin(x)}{2y + 1} \quad y(0) = 2 \]
satisfies
(a) \( 2y + 1 = 6 - e^{-\cos x} \)  
(b) \( y^2 + y = 7 - \cos x \)
(c) \( y^2 + y = 6 \cos x \)  
(d) \( 2y + 1 = 5e^{-\cos x} \)
(e) \( e^{2y+1} = e^5 + \arcsin x \)

14. (6 pts.) The solution to the initial value problem
\[ \frac{dy}{dx} + xy + x = 0 \quad y(0) = 0 \]
is
(a) \( y = e^{-\frac{x^2}{2}} - 1 \)  
(b) \( y = e - e^{-\frac{x^2}{2}+1} \)  
(c) \( y = xe^x \)
(d) \( y = 1 - e^{-x} \)  
(e) \( y = e^{-x} - 1 \)
15. (6 pts.) Investigate the convergence or divergence of the sequence
\[ \lim_{n \to \infty} (-1)^n \frac{3n^2}{n^2 + 1}. \]
If the sequence converges, find its limit.

(a) $-3$  
(b) $(-1)^n 3$  
(c) The sequence is divergent  
(d) $3$  
(e) $0$

16. (6 pts.) Investigate convergence or divergence of the series
\[ \sum_{n=1}^{\infty} (-1)^{n-1} \frac{(4 - \pi)^{n-1}}{\pi^n}. \]
If the series converges, calculate its sum. Note: $4 > \pi > 3$.

(a) $\frac{\pi}{4}$  
(b) $-\frac{\pi}{4}$  
(c) The series is divergent  
(d) $\frac{1}{4}$  
(e) $-\frac{1}{4}$
17. (6 pts.) The series
\[ \sum_{n=1}^{\infty} \frac{1}{n^{3/2}} \cos \left( \frac{1}{n} \right) \]
is
(a) absolutely convergent by limit comparison test with \( \sum_{n=1}^{\infty} \frac{1}{n^{3/2}} \)
(b) conditionally convergent by root test
(c) divergent by integral test
(d) divergent by comparison with \( \sum_{n=1}^{\infty} \frac{1}{n^{3/2}} \)
(e) absolutely convergent by ratio test

18. (6 pts.) Which of the following series converge conditionally?

(1) \( \sum_{n=0}^{\infty} \frac{(-1)^n}{\sqrt{n} + 1} \);  (2) \( \sum_{n=0}^{\infty} \frac{1}{\sqrt{n} + 1} \);  (3) \( \sum_{n=0}^{\infty} (-1)^n \frac{1}{n^{5/2} + 1} \).

(a) (1) and (2) converge conditionally, (3) does not converge conditionally
(b) (2) converges conditionally, (1) and (3) do not converge conditionally
(c) (1) converges conditionally, (2) and (3) do not converge conditionally
(d) (1) and (3) converge conditionally, (2) does not converge conditionally
(e) (3) converges conditionally, (1) and (2) do not converge conditionally
19. (6 pts.) The series
\[ \sum_{n=1}^{\infty} \frac{8^n}{n^2} (x - 1)^{3n} \]
has the radius of convergence

(a) 0  (b) \frac{1}{2}  (c) \infty  (d) 1  (e) \frac{1}{8}

20. (6 pts.) Consider the Taylor series of
\[ f(x) = \sum_{n=1}^{\infty} \frac{n^n}{n!} x^n. \]
Find \( f^{(100)}(0) \).

(a) \frac{100^{100}}{((100)!)^2}  (b) \frac{(100)!}{100^{100}}  (c) \frac{100^{100}}{(100)!}

(d) (100)!  (e) 100^{100}
21. (6 pts.) Determine for which \( x \) the approximation of \( \sin x \) by its 3rd degree MacLaurin polynomial \( T_3(x) \) (Taylor polynomial centered at 0) is accurate to within \( \frac{1}{3840} \), by using the Alternating Series Remainder Estimation Theorem. Note: \( 3840 = 120 \cdot 2^5 \).

\begin{align*}
(a) & -\sqrt{32} < x < \sqrt{32} & (b) & -1 < x < 1 \\
(c) & -\sqrt{120} < x < \sqrt{120} & (d) & -120 < x < 120 \\
(e) & -\frac{1}{2} < x < \frac{1}{2}
\end{align*}

22. (6 pts.) Let \( x = \sin(9t) \) and \( y = \cos(9t) \). Then \( \frac{dy}{dx} = 
\begin{align*}
(a) & \tan(9t) & (b) & -\tan(9t) & (c) & 9\tan(t) \\
(d) & 81\sec^2(9t) & (e) & \cot(9t)
\end{align*}
23. (6 pts.) The point \((2, \frac{13\pi}{6})\) in polar coordinates corresponds to which point below in Cartesian coordinates?

(a) \((\sqrt{3}, 1)\)

(b) \((-\sqrt{3}, 1)\)

(c) \((1, \sqrt{3})\)

(d) \((-1, \sqrt{3})\)

(e) Since \(\frac{13\pi}{6} > 2\pi\), there is no such point.

24. (6 pts.) Which integral below gives the surface area of the surface of revolution obtained by rotating the polar curve \(r = \sin \theta\), \(0 \leq \theta \leq \pi\) about the \(x\)-axis?

**Hint:** A polar curve is also a parameterized curve.

(a) \(2\pi \int_0^\pi \sin \theta \cos^2 \theta \, d\theta\)

(b) \(2\pi \int_0^\pi \cos^2 \theta \, d\theta\)

(c) \(2\pi \int_0^\pi \sin^2 \theta \, d\theta\)

(d) \(\frac{\pi}{2} \int_0^\pi \sin \theta \cos^2 \theta \, d\theta\)

(e) \(2\pi \int_0^\pi \sin \theta \cos \theta \, d\theta\)
25. (6 pts.) Find the area inside the cardioid $r = 1 + \sin \theta$.

(a) $\frac{3}{2}$  (b) $2\pi$  (c) $2$  (d) $3\pi + \ln 4$  (e) $\frac{3\pi}{2}$
BE SURE THAT YOU HAVE ALL 14 PAGES OF THE TEST.
NO CALCULATORS ARE TO BE USED.
The exam lasts for two hours.
When told to begin, remove this answer sheet and keep it under the
rest of your test. When told to stop, hand in just this one page.
The Honor Code is in effect for this examination, including keeping your answer
sheet under cover.

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1. (●) (b) (c) (d) (e)
2. (a) (b) (●) (d) (e)
3. (●) (b) (c) (d) (e)
4. (a) (b) (●) (d) (e)
5. (a) (b) (c) (●) (e)
6. (●) (b) (c) (d) (e)
7. (a) (●) (c) (d) (e)
8. (a) (b) (●) (d) (e)
9. (a) (b) (c) (●) (e)
10. (a) (b) (c) (●) (e)
11. (a) (b) (c) (d) (●)
12. (a) (b) (c) (d) (●)
13. (a) (●) (c) (d) (e)
14. (●) (b) (c) (d) (e)
15. (a) (b) (●) (d) (e)
16. (a) (b) (c) (●) (e)
17. (●) (b) (c) (d) (e)
18. (a) (b) (●) (d) (e)
19. (a) (●) (c) (d) (e)
20. (a) (b) (c) (d) (●)
21. (a) (b) (c) (d) (●)
22. (a) (●) (c) (d) (e)
23. (●) (b) (c) (d) (e)
24. (a) (b) (●) (d) (e)
25. (a) (b) (c) (d) (●)