The Honor Code is in effect for this examination. All work is to be your own.

No calculators.

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Be sure that you have all 9 pages of the test.

Please mark your answers with an X, not a circle!

1. (a) (b) (c) (d) (e)
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10. (a) (b) (c) (d) (e)

Please do NOT write in this box.

Multiple Choice   

11.  
12.  
13.  

Total  

Multiple Choice

1. (7 pts.) Calculate
\[ \lim_{n \to \infty} \frac{(\ln n)^2}{n}. \]
(a) 1, (b) \( \infty \), (c) does not exist, (d) \( e^2 \), (e) 0

2. (7 pts.) The series
\[ \sum_{n=2}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}} \]
(a) diverges even though \( \lim_{n \to \infty} \frac{(-1)^{n+1}}{\sqrt{n}} = 0 \).
(b) does not converge absolutely but does converge conditionally.
(c) diverges because \( \lim_{n \to \infty} \frac{(-1)^{n+1}}{\sqrt{n}} \neq 0 \).
(d) converges absolutely.
(e) diverges because the terms alternate.
3. (7 pts.) Use Comparison Tests to determine which one of the following series is divergent.

(a) \[ \sum_{n=1}^{\infty} \frac{1}{n^3 + 1} \]

(b) \[ \sum_{n=1}^{\infty} \frac{1}{n^2 + 8} \]

(c) \[ \sum_{n=1}^{\infty} \frac{n^2 - 1}{n^3 + 100} \]

(d) \[ \sum_{n=1}^{\infty} \frac{n}{n + 1} \left( \frac{1}{2} \right)^n \]

(e) \[ \sum_{n=1}^{\infty} 7 \left( \frac{5}{6} \right)^n \]

4. (7 pts.) Consider the following series

(I) \[ \sum_{n=1}^{\infty} \left( \frac{2n^2 + 7}{n^2 + 1} \right)^n \]

(II) \[ \sum_{n=2}^{\infty} \frac{2^{1/n}}{n - 1} \]

(III) \[ \sum_{n=1}^{\infty} \frac{n!}{e^n} \]

Which of the following statements is true?

(a) (I) converges, (II) diverges, and (III) converges.

(b) (I) diverges, (II) diverges, and (III) converges.

(c) (I) converges, (II) diverges, and (III) diverges.

(d) They all diverge.

(e) They all converge.
5. (7 pts.) Which series below conditionally converges?

(a) \[ \sum_{n=1}^{\infty} (-1)^{n-1} \]
(b) \[ \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\sqrt{n}} \]
(c) \[ \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^2} \]
(d) \[ \sum_{n=1}^{\infty} \frac{(-1)^{n-1} e^n}{\sqrt{n}} \]
(e) \[ \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\sqrt{n^3}} \]

6. (7 pts.) Consider the following series

(I) \[ \sum_{n=1}^{\infty} \frac{2^n}{n!} \]
(II) \[ \sum_{n=1}^{\infty} \left( \frac{n^2 + n}{2n^2 + 1} \right)^n \]

Which of the following statements is true?

(a) They both converge.
(b) The Ratio Test applied to (I) is inconclusive.
(c) (I) converges and (II) diverges.
(d) (I) diverges and (II) converges.
(e) They both diverge.
7. (7 pts.) Which series below is the MacLaurin series (Taylor series centered at 0) for \( \frac{x^2}{1 + x} \)?

(a) \[ \sum_{n=0}^{\infty} \frac{x^{n+2}}{n + 2} \]

(b) \[ \sum_{n=2}^{\infty} \frac{(-1)^n x^{2n-2}}{n!} \]

(c) \[ \sum_{n=0}^{\infty} (-1)^n x^{2n} \]

(d) \[ \sum_{n=0}^{\infty} x^{2n+2} \]

(e) \[ \sum_{n=0}^{\infty} (-1)^n x^{n+2} \]

8. (7 pts.) Which series below is a power series for \( \cos(\sqrt{x}) \)?

(a) \[ \sum_{n=0}^{\infty} \frac{(-1)^n x^n}{(2n + 1)!} \]

(b) \[ \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n - \frac{1}{2}}}{(2n)!} \]

(c) \[ \sum_{n=0}^{\infty} \frac{(-1)^n x^n}{(2n)!} \]

(d) \[ \sum_{n=0}^{\infty} \frac{(-1)^n \sqrt{x}^n}{(2n)!} \]

(e) \[ \sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n^2 + 1} \]
9. (7 pts.) Calculate
\[ \lim_{x \to 0} \frac{\sin(x^3) - x^3}{x^9}. \]

**Hint:** Without MacLaurin series this may be a long problem.

(a) \( \infty \) \hspace{1cm} (b) \( \frac{9}{7} \) \hspace{1cm} (c) 0 \hspace{1cm} (d) \( -\frac{1}{6} \) \hspace{1cm} (e) \( \frac{7}{9} \)

10. (7 pts.) The following is the fifth order Taylor polynomial of the function \( f(x) \) at \( a \)
\[ T_5(x) = 2 - 2(x - a) + \sqrt{5}(x - a)^2 - \frac{\pi}{2}(x - a)^3 + (x - a)^4 + 13(x - a)^5 \]

What is \( f^{(3)}(a) \)?

(a) \( -\frac{\pi}{2} \) \hspace{1cm} (b) \( \sqrt{5} \) \hspace{1cm} (c) \( -3\pi \)

(d) 24 \hspace{1cm} (e) \( 2\sqrt{5} \)
11. (11 pts.) Does the series
\[ \sum_{n=1}^{\infty} \frac{(n!)^n}{n^{2n}} \]
converge or diverge? Show your reasoning and state clearly any theorems or tests you are using.

Remark: The correct answer with no justification is worth 2 points.
12. (11 pts.) Find the radius of convergence and interval of convergence of the power series

\[ \sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}} (x - 3)^n \]

**Remark:** The correct answer with no justification is worth 2 points.
13. (11 pts.)
(a) Show that

\[ \sum_{n=0}^{\infty} (-1)^n x^{2n} = \frac{1}{1 + x^2} \]

provided that \(|x| < 1\).

(b) Find

\[ \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n + 1)(\sqrt{3})^{2n+1}} \]

(Hint: First use term-by-term integration on the series in part (a).)
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Multiple Choice

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Total _____________