Math 10560, Practice Exam 1.
February 14, 2012

- The Honor Code is in effect for this examination. All work is to be your own.
- No calculators.
- The exam lasts for 1 hour and 15 min.
- Be sure that your name is on every page in case pages become detached.
- Be sure that you have all 10 pages of the test.

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

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Multiple Choice

9. 
10. 
11. 
12. 
Total
Multiple Choice

1. (7 pts.) Simplify the following expression for $x$.

$$x = \log_3 81 + \log_3 \frac{1}{9}.$$ 

(a) $x = 9$  
(b) $x = 6$  
(c) $x = \ln 9 - \ln 3$

(d) $x = \ln 3$  
(e) $x = 2$

2. (7 pts.) The function $f(x) = x^3 + 3x + e^{2x}$ is one-to-one. Compute $(f^{-1})'(1)$.

(a) 0  
(b) $\frac{1}{5}$  
(c) $\frac{1}{4}$  
(d) $\frac{1}{6+e}$  
(e) $\frac{1}{6+2e}$
3. (7 pts.) Differentiate the function

\[ f(x) = \frac{(x^2 - 1)^4}{\sqrt{x^2 + 1}}. \]

(a) \( f'(x) = \frac{x(x^2 - 1)^4}{\sqrt{x^2 + 1}} \left( \frac{8}{x^2 - 1} + \frac{1}{x^2 + 1} \right) \)

(b) \( f'(x) = \frac{(x^2 - 1)^4}{\sqrt{x^2 + 1}} \left( \frac{4}{x^2 - 1} - \frac{1}{x^2 + 1} \right) \)

(c) \( f'(x) = \frac{(x^2 - 1)^4}{\sqrt{x^2 + 1}} \left( \frac{4}{x^2 - 1} + \frac{1}{x^2 + 1} \right) \)

(d) \( f'(x) = \frac{(x^2 - 1)^4}{\sqrt{x^2 + 1}} \left( \frac{8}{x^2 - 1} - \frac{1}{x^2 + 1} \right) \)

(e) \( f'(x) = \frac{x(x^2 - 1)^4}{\sqrt{x^2 + 1}} \left( \frac{8}{x^2 - 1} - \frac{1}{x^2 + 1} \right) \)

4. (7 pts.) Compute the integral

\[ \int_{2e}^{2e} \frac{1}{x(\ln \frac{x}{2})^2} \, dx. \]

(a) 2 (b) \( \frac{3}{2} \) (c) \( \frac{1}{2} \) (d) 1 (e) 0
5. (7 pts.) Which of the following expressions gives the partial fraction decomposition of the function 

\[ f(x) = \frac{x^2 - 2x + 6}{x^3(x - 3)(x^2 + 4)} \]?

(a) \[ \frac{A}{x^3} + \frac{B}{x - 3} + \frac{C}{x^2 + 4} \]

(b) \[ \frac{A}{x^3} + \frac{B}{x - 3} + \frac{Cx + D}{x^2 + 4} \]

(c) \[ \frac{A}{x^3} + \frac{B}{x^2} + \frac{C}{x} + \frac{D}{x - 3} + \frac{E}{x^2 + 4} \]

(d) \[ \frac{A}{x^3} + \frac{B}{x^2} + \frac{C}{x} + \frac{D}{x - 3} + \frac{Ex + F}{x^2 + 4} \]

(e) \[ \frac{A}{x^3} + \frac{B}{x^2} + \frac{C}{x} + \frac{D}{x - 3} + \frac{E}{x + 2} + \frac{F}{x - 2} \]

6. (7 pts.) Find \( f'(x) \) if 

\[ f(x) = x^\ln x \].

(a) \[ 2(\ln x)x^{\ln x} \]

(b) \[ x^{\ln x}\ln x \]

(c) \[ 2(\ln x)x^{(\ln x)-1} \]

(d) \[ x^{\ln x}(\ln x + 1) \]

(e) \[ x^{(\ln x)-1}\ln x \]
7. (7 pts.) Calculate the following integral.

\[ \int_{0}^{1} \frac{\arctan x}{1 + x^2} \, dx . \]

(a) \( \frac{1}{2} \)  (b) \( \frac{\pi}{8} \)  (c) \( \frac{\pi^2}{32} \)  (d) \( \ln 2 \)  (e) \( \frac{\pi^2}{8} \)

8. (7 pts.) Evaluate the integral

\[ \int_{0}^{\pi/2} \sin^3(x) \cos^5(x) \, dx. \]

(a) \( 0 \)  (b) \( \frac{\pi}{2} \)  (c) \( -\frac{1}{24} \)  (d) \( \frac{1}{24} \)  (e) \( \frac{1}{4} \)
Partial Credit
You must show your work on the partial credit problems to receive credit!

9. (11 pts.) Compute the limit
\[
\lim_{x \to 2} \left( \frac{x}{2} \right) \frac{1}{x - 2}.
\]
10. (11 pts.) Evaluate the integral
\[ \int x^2 \cos(2x) \, dx. \]
11. (11 pts.) Evaluate:

$$\int \frac{1}{3} x^3 \sqrt{9 - x^2} \, dx.$$
12. (11 pts.) Let $C(t)$ be the concentration of a drug in the bloodstream. As the body eliminates the drug, $C(t)$ decreases at a rate that is proportional to the amount of the drug that is present at the time. Thus $C'(t) = kC(t)$, where $k$ is a constant. The initial concentration of the drug is 4 mg/ml. After 5 hours, the concentration is 3 mg/ml.

(a) Give a formula for the concentration of the drug at time $t$.

(b) How much drug will there be in 10 hours?

(c) How long will it take for the concentration to drop to 0.5 mg/ml?
The following is the list of useful trigonometric formulas:

$$\sin^2 x + \cos^2 x = 1$$

$$1 + \tan^2 x = \sec^2 x$$

$$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$$

$$\cos^2 x = \frac{1}{2}(1 + \cos 2x)$$

$$\sin 2x = 2 \sin x \cos x$$

$$\sin x \cos y = \frac{1}{2}(\sin(x - y) + \sin(x + y))$$

$$\sin x \sin y = \frac{1}{2}(\cos(x - y) - \cos(x + y))$$

$$\cos x \cos y = \frac{1}{2}(\cos(x - y) + \cos(x + y))$$

$$\int \sec \theta = \ln |\sec \theta + \tan \theta| + C$$

$$\int \csc \theta = \ln |\csc \theta - \cot \theta| + C$$
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