Name: ______________________________

Instructor: __________________________

Math 10560, Practice Exam 2.
March 14, 2012

- The Honor Code is in effect for this examination. All work is to be your own.
- No calculators.
- The exam lasts for 1 hour and 15 min.
- Be sure that your name is on every page in case pages become detached.
- Be sure that you have all 12 pages of the test.

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1.	(a)	(b)	(c)	(d)	(e)
2.	(a)	(b)	(c)	(d)	(e)
3.	(a)	(b)	(c)	(d)	(e)
4.	(a)	(b)	(c)	(d)	(e)
5.	(a)	(b)	(c)	(d)	(e)
6.	(a)	(b)	(c)	(d)	(e)
7.	(a)	(b)	(c)	(d)	(e)
8.	(a)	(b)	(c)	(d)	(e)
9.	(a)	(b)	(c)	(d)	(e)
10.	(a)	(b)	(c)	(d)	(e)

Please do NOT write in this box.

Multiple Choice	________
11.	________
12.	________
13.	________
14.	________
Total	________

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Multiple Choice

1.(6 pts.) Evaluate the improper integral

$$\int_4^\infty \frac{1}{(x-2)(x-3)}\, dx.$$

(a) $\ln 3$ (b) $\ln \frac{1}{2}$ (c) $\ln 2$

(d) the integral diverges (e) $3\ln 2$

2.(6 pts.) What can be said about the integrals

$$(i) \int_0^1 \frac{e^x}{x^2}\, dx;$$

$$(ii) \int_1^\infty \frac{\cos^2 x}{x^2}\, dx?$$

(a) both (i) and (ii) converge

(b) both (i) and (ii) diverge

(c) (i) converges and (ii) diverges

(d) (i) diverges and (ii) converges

(e) neither integral (i) nor (ii) is improper

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3.(6 pts.) Find the centroid of the triangle with vertices $(-1, 0)$, $(1, 0)$ and $(0, 3)$.

(a) $(0, \frac{4}{3})$ (b) $(0, \frac{3}{2})$

(c) $(0, 1)$ (d) $(0, 2)$

(e) $(0, 3)$

4.(6 pts.) Use Euler's method with step size 0.1 to estimate $y(1.2)$ where $y(x)$ is the solution to the initial value problem

$$y' = xy + 1 \qquad y(1) = 0.$$

(a) $y(1.2) \approx .201$ (b) $y(1.2) \approx .112$ (c) $y(1.2) \approx .111$

(d) $y(1.2) \approx .101$ (e) $y(1.2) \approx .211$

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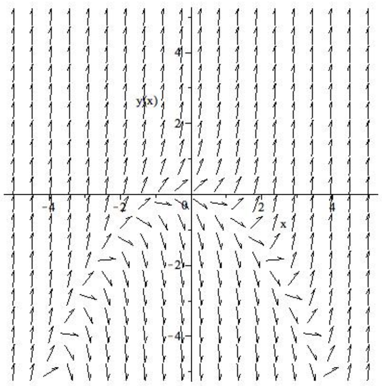
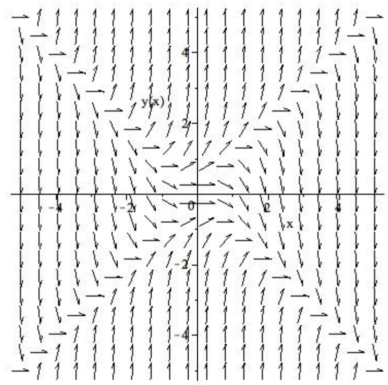
5.(6 pts.) Which of the following gives the direction field for the differential equation

$$y' = y^2 - x^2$$

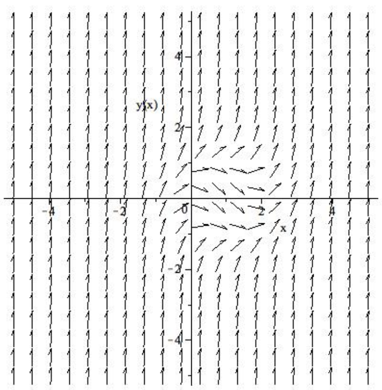
Note the letter corresponding to each graph is at the lower left of the graph.



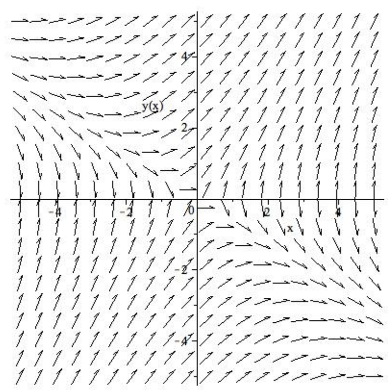
(a)



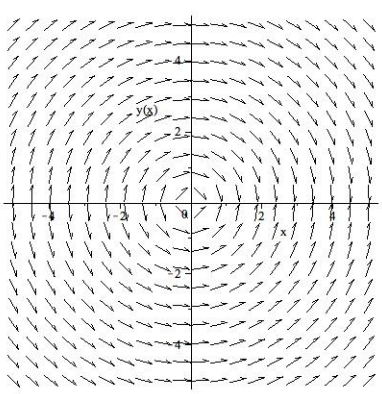
(b)



(c)



(d)



(e)

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6.(6 pts.) Find the solution of the differential equation

$$\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{1+x^2}$$

with initial condition $y(0) = 0$.

(a) $y = \dfrac{1}{\sqrt{1+x^2}}$ (b) $y = \dfrac{x}{1+x}$ (c) $y = \dfrac{x^2}{\sqrt{1+x^2}}$

(d) $y = \dfrac{x}{1+x^2}$ (e) $y = \dfrac{x}{\sqrt{1+x^2}}$

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7.(6 pts.) Find a general solution, valid for $-\frac{\pi}{2} < x < \frac{\pi}{2}$, of the differential equation

$$\frac{dy}{dx} - (\tan x)y = 1.$$

(a) $y = \dfrac{x + \sin x + C}{\sin x}$ (b) $y = \dfrac{\sin x + C}{\cos x}$ (c) $y = \dfrac{x + \sin x + C}{\cos x}$

(d) $y = \dfrac{\cos x + C}{\sin x}$ (e) $y = \tan x + \cos x + C$

8.(6 pts.) Solve the differential equation

$$8\frac{d^2y}{dt^2} - 4\frac{dy}{dt} + y = 0$$

is given by

(a) $y(t) = c_1 e^{0.25t} + c_2 e^{-0.25t}$ (b) $y(t) = (c_1 \cos(\frac{1}{4}t) + c_2 \sin(\frac{1}{4}t))$

(c) $y(t) = e^{0.25t}(c_1 \cos(\frac{1}{4}t) + c_2 \sin(\frac{1}{4}t))$ (d) $y(t) = e^t(c_1 \cos(t) + c_2 \sin(t))$

(e) $y(t) = c_1 e^{0.25t} + c_2 t e^{0.25t}$

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9.(6 pts.) Determine which of the following statements are true for the following differential equation:

$$y'' - y' - 6y = 3x.$$

I: $y_c = c_1e^{3x} + c_2e^{-2x}$ is the solution to the corresponding homogeneous equation.
II: $y_c = c_1e^{6x} + c_2e^{-x}$ is the solution to the corresponding homogeneous equation.
III: $y_p = -\frac{x}{2} + \frac{1}{2}$ is a particular solution.
IV: $y_p = \dfrac{x+1}{2}$ is a particular solution.

(a) I and IV

(b) II and IV

(c) I and III

(d) none of the above

(e) II and III

10.(6 pts.) Find the limit: $\lim\limits_{n\to\infty} \dfrac{(-1)^n n^2}{n^2+n+1}$

(a) -1

(b) 0

(c) 2

(d) Does not exist

(e) 1

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Partial Credit

You must show your work on the partial credit problems to receive credit!

11.(10 pts.) Solve the initial value problem

$$y'' + y' - 2y = 0, \quad y(0) = 1, \; y'(0) = 2.$$

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12.(10 pts.) Calculate the arc length of the curve if $y = \dfrac{x^2}{4} - \ln(\sqrt{x})$, where $2 \leq x \leq 4$.

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13.(10 pts.) (a) Circle the letter below alongside the trapezoidal approximation to

$$\ln 3 = \int_1^3 \frac{1}{x} dx \quad \text{using} \quad n = 8$$

A $\displaystyle\int_1^3 \frac{1}{x} dx \approx \frac{1}{8}\Big[1 + 2\Big(\frac{4}{5}\Big) + 2\Big(\frac{2}{3}\Big) + 2\Big(\frac{4}{7}\Big) + 2\Big(\frac{1}{2}\Big) + 2\Big(\frac{4}{9}\Big) + 2\Big(\frac{2}{5}\Big) + 2\Big(\frac{4}{11}\Big) + \Big(\frac{1}{3}\Big)\Big]$

B $\displaystyle\int_1^3 \frac{1}{x} dx \approx \frac{1}{12}\Big[1 + 4\Big(\frac{4}{5}\Big) + 2\Big(\frac{2}{3}\Big) + 4\Big(\frac{4}{7}\Big) + 2\Big(\frac{1}{2}\Big) + 4\Big(\frac{4}{9}\Big) + 2\Big(\frac{2}{5}\Big) + 4\Big(\frac{4}{11}\Big) + \Big(\frac{1}{3}\Big)\Big]$

C $\displaystyle\int_1^3 \frac{1}{x} dx \approx \frac{1}{8}\Big[1 + \Big(\frac{4}{5}\Big) + \Big(\frac{2}{3}\Big) + \Big(\frac{4}{7}\Big) + \Big(\frac{1}{2}\Big) + \Big(\frac{4}{9}\Big) + \Big(\frac{2}{5}\Big) + \Big(\frac{4}{11}\Big) + \Big(\frac{1}{3}\Big)\Big]$

(b) Recall that the error E_T in the trapezoidal rule for approximating $\int_a^b f(x)dx$ satisfies

$$\Big|\int_a^b f(x)dx - T_n\Big| = |E_T| \leq \frac{K(b-a)^3}{12n^2}$$

whenever $|f''(x)| \leq K$ for all $a \leq x \leq b$.

Use the above error bound to determine a value of n for which the trapezoidal approximation to $\ln 3 = \displaystyle\int_1^3 \frac{1}{x} dx$ has an error

$$|E_T| \leq \frac{1}{3} 10^{-4}.$$

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14.(10 pts.) Find the family of orthogonal trajectories to the family of curves given by

$$y = kx^2.$$

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The following is the list of useful trigonometric formulas:

$$\sin^2 x + \cos^2 x = 1$$
$$1 + \tan^2 x = \sec^2 x$$

$$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$$

$$\cos^2 x = \frac{1}{2}(1 + \cos 2x)$$

$$\sin 2x = 2 \sin x \cos x$$

$$\sin x \cos y = \frac{1}{2}\big(\sin(x - y) + \sin(x + y)\big)$$

$$\sin x \sin y = \frac{1}{2}\big(\cos(x - y) - \cos(x + y)\big)$$

$$\cos x \cos y = \frac{1}{2}\big(\cos(x - y) + \cos(x + y)\big)$$