Name: ______________________________

Math 10560, Practice Final Exam: Instructor: __________________________
May 1, 2012

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14.	(a)	(b)	(c)	(d)	(e)						

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Multiple Choice

1.(6 pts.) Let $f(x) = e^x - 1$ and let f^{-1} denote the inverse function. Then $(f^{-1})'(e^2 - 1) =$ is

(a) e^{-1}

(b) $\dfrac{1}{e^2 - 1}$

(c) e

(d) e^2

(e) e^{-2}

2.(6 pts.) Solve the following equation for x:

$$\ln(x+4) - \ln x = 1 \ .$$

(a) $x = \dfrac{4}{1-e}$

(b) $x = \dfrac{4}{e-1}$ and $x = \dfrac{4}{e+1}$

(c) There is no solution.

(d) $x = e + 2$ and $x = e - 2$

(e) $x = \dfrac{4}{e-1}$

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3.(6 pts.) Find the derivative of $(x^2+1)^{x^2+1}$.

(a) $(x^2+1)^{x^2+1}\big(2x\ln(x^2+1)\big)$

(b) $(x^2+1)^{x^2+1}\ 2x\big(\ln(x^2+1)+1\big)$

(c) $2x(x^2+1)^{x^2}$

(d) $(x^2+1)^{x^2+1}$

(e) This function is not defined and hence has no derivative.

4.(6 pts.) $\lim_{x\to 0^+} (\cos x)^{\frac{1}{x^2}} =$

(a) $e^{-\frac{1}{2}}$

(b) 1

(c) Does not exist

(d) ∞

(e) e

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5.(6 pts.) The integral

$$\int_0^{\pi/2} x\cos(x)dx$$

is

(a) divergent (b) $\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}$ (c) $\frac{\pi}{2} - 1$

(d) 0 (e) $1 - \frac{\pi}{2}$

6.(6 pts.) Evaluate

$$\int \frac{x^2}{\sqrt{9-x^2}}\,dx.$$

(a) $\frac{9}{2}\left[\arcsin(x/3) - \frac{x}{3}\right] + C$ (b) $\frac{1}{2}x\sqrt{9-x^2} + C$

(c) $9\arcsin(x/3) + C$ (d) $\frac{9}{2}\left[\arcsin(x/3) - \frac{x\sqrt{9-x^2}}{9}\right] + C$

(e) $\frac{9}{2}\left[\arcsin(x/3) - \frac{x^2}{9}\right] + C$

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7.(6 pts.) If you expand $\dfrac{2x+1}{x^3+x}$ as a partial fraction, which expression below would you get?

(a) $\dfrac{1}{x} + \dfrac{-x+2}{x^2+1}$

(b) $\dfrac{-1}{x^2} + \dfrac{1}{x+1}$

(c) $\dfrac{2}{x} + \dfrac{1}{x^2+1}$

(d) $\dfrac{-1}{x} + \dfrac{x}{x^2+1}$

(e) $\dfrac{-2}{x} + \dfrac{1}{x^2+1}$

8.(6 pts.) The integral

$$\int_0^2 \frac{1}{1-x}dx$$

is

(a) $\dfrac{\pi}{\sqrt{2}}$

(b) 0

(c) divergent

(d) $\dfrac{\pi}{6}$

(e) $\ln 2$

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9.(6 pts.) The moment about the y–axis of the plane region of density ρ lying above the parabola $y = x^2$ and below the line $y = 9$ is given by the integral

(a) $\rho \int_0^9 \frac{y^2}{2}\,dy$

(b) $\rho \int_{-3}^3 x(9 - x^2)\,dx$

(c) $\rho \int_0^9 y(9 - y)\,dy$

(d) $4\rho \int_0^3 x(9 - x^2)\,dx$

(e) $\rho \int_{-3}^3 \frac{(9 - x^2)^2}{2}\,dx$

10.(6 pts.) If 100 grams of radioactive material with a half–life of two days are present at day zero, how many grams are left at day three?

(a) $\frac{100}{2^{1/3}}$ (b) 50 (c) $\frac{100}{\sqrt{2}}$ (d) $\frac{100}{4^{1/3}}$ (e) $\frac{100}{\sqrt{8}}$

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11.(6 pts.) If $x\dfrac{dy}{dx} + 3y = \dfrac{4}{x}$, and $y(1) = 10$, find $y(2)$.

(a) 2 (b) $\dfrac{1}{2}$ (c) 7 (d) $\dfrac{4}{3}$ (e) 0

12.(6 pts.) The solution to the initial value problem

$$y' = x\cos^2 y \qquad y(2) = 0$$

satisfies the implicit equation

(a) $\cos y = x - 1$

(b) $\dfrac{ey}{2} = e^{\cos x} - e^{\cos 2}$

(c) $\tan(y) = \dfrac{x^2}{2} - 2$

(d) $\cos(y) = x + \cos(2)$

(e) $e^{2y+1} = \arcsin(x-2) + e$

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13.(6 pts.) The general solution to the differential equation

$$\frac{d^2y}{dt^2} - 3\frac{dy}{dt} + 2y = 0$$

is given by

(a) $y(t) = c_1 e^{2t} + c_2 e^t$

(b) $y(t) = c_1 e^t + c_2 t e^{-t}$

(c) $y(t) = c_1 \cos(2t) + c_2 \sin(t)$

(d) $y(t) = c_1 e^{2t} + c_2 t e^t$

(e) $y(t) = c_1 e^{-2t} + c_2 e^t$

14.(6 pts.) Find $\displaystyle\sum_{n=1}^{\infty} \frac{2^{2n}}{3 \cdot 5^{n-1}}$

(a) $\dfrac{5}{12}$ (b) $\dfrac{5}{4}$ (c) $\dfrac{20}{3}$ (d) $\dfrac{5}{3}$ (e) $\dfrac{4}{15}$

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15.(6 pts.) Which of the following series converge conditionally?

$$\text{(I)} \sum_{n=0}^{\infty} \frac{(-1)^n}{n^2} \qquad \text{(II)} \sum_{n=2}^{\infty} \frac{(-1)^n\, n}{\ln n} \qquad \text{(III)} \sum_{n=0}^{\infty} \frac{(-1)^n}{n} \;?$$

(a) (I) and (III) converge conditionally, (II) does not converge conditionally

(b) (I) and (II) converge conditionally, (III) does not converge conditionally

(c) (II) converges conditionally, (I) and (III) do not converge conditionally

(d) (III) converges conditionally, (I) and (II) do not converge conditionally

(e) (II) and (III) converge conditionally, (I) does not converge conditionally

16.(6 pts.) Which series below absolutely converges?

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^3}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\ln(n+1)}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n^3}}{n^2+1}$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n n!}{n^3}$

(e) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}\pi^n}{3^n}$

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17.(6 pts.) The interval of convergence of the series

$$\sum_{n=1}^{\infty} \frac{(x+3)^n}{\sqrt{n}}$$

is

(a) $[2,4]$ (b) $[-4,-2)$ (c) $(-4,-2)$

(d) $(2,4)$ (e) $(-1,1)$

18.(6 pts.) If $f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n (x-2)^n}{(2n+1)!}$, find the power series centered at 2 for the function $\int_2^x f(t)\,dt$.

(a) The given function can not be represented by a power series centered at 2.

(b) $\sum_{n=0}^{\infty} \frac{(-1)^n (x-2)^{n+1}}{(n+1)!}$

(c) $\sum_{n=0}^{\infty} \frac{(-1)^n (x-2)^{n+1}}{(n+1)(2n+1)!}$

(d) $\sum_{n=0}^{\infty} \frac{(-1)^n (x-2)^{2n+1}}{(n+1)(2n)!}$

(e) $\sum_{n=0}^{\infty} \frac{(-1)^n (x-2)^{n+1}}{(n^2)(2n+1)!}$

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19.(6 pts.) Which series below is the MacLaurin series (Taylor series centered at 0) for $\dfrac{x^2}{1+x}$?

(a) $\displaystyle\sum_{n=0}^{\infty}(-1)^n x^{2n}$ (b) $\displaystyle\sum_{n=2}^{\infty}\frac{(-1)^n x^{2n-2}}{n!}$ (c) $\displaystyle\sum_{n=0}^{\infty} x^{2n+2}$

(d) $\displaystyle\sum_{n=0}^{\infty}(-1)^n\ x^{n+2}$ (e) $\displaystyle\sum_{n=0}^{\infty}\frac{x^{n+2}}{n+2}$

20.(6 pts.) Find the degree 3 MacLaurin polynomial (Taylor polynomial centered at 0) for the function

$$\frac{e^x}{1-x^2}$$

(a) $1+x-\dfrac{x^3}{6}$ (b) $1-\dfrac{x^2}{2}+\dfrac{x^3}{5}$

(c) $1+x-\dfrac{5x^3}{3}$ (d) $1+x+\dfrac{3x^2}{2}+\dfrac{7x^3}{6}$

(e) $1+x+\dfrac{x^2}{6}+0\,x^3$

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21.(6 pts.) $\lim_{x\to 0} \dfrac{\sin(x^3) - x^3}{x^9} =$

Hint: Without MacLaurin series this may be a long problem.

(a) 0 (b) ∞ (c) $\dfrac{9}{7}$ (d) $\dfrac{7}{9}$ (e) $-\dfrac{1}{6}$

22.(6 pts.) Which integral below gives the arclength of the curve $x = 1 - 2\cos t$, $y = \sin^2\big(t/2\big)$, $0 \le t \le 2\pi$?

(a) $\displaystyle\int_0^{2\pi} \sqrt{1 - 2\cos(t) + \cos^2(t) + \sin^2(t/2)\cos^2(t/2)}\ dt$

(b) $\displaystyle\int_0^{2\pi} \sqrt{4\sin^2 t + \sin^2(t/2)\cos^2(t/2)}\ dt$

(c) $\displaystyle\int_0^{2\pi} \sqrt{4\sin^2 t + \sin^4(t/2)}\ dt$

(d) $\displaystyle\int_0^{2\pi} \sqrt{\sin^2(t/2) - 2\sin^2(t/2)\cos(t)}\ dt$

(e) $\displaystyle\int_0^{2\pi} \sqrt{1 - 2\cos(t) + \cos^2(t) + \sin^4(t/2)}\ dt$

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23.(6 pts.) The point $(2, \frac{11\pi}{3})$ in polar coordinates corresponds to which point below in Cartesian coordinates?

(a) $(1, -\sqrt{3})$

(b) $(\sqrt{3}, -1)$

(c) $(-\sqrt{3}, 1)$

(d) $(-1, \sqrt{3})$

(e) Since $\frac{11\pi}{3} > 2\pi$, there is no such point.

24.(6 pts.) Find the equation for the tangent line to the curve with polar equation: $r = 2 - 2\cos\theta$ at the point $\theta = \pi/2$.

(a) $y = 2 - \pi + 2x$

(b) $y = 2 + \frac{\pi}{2} - x$

(c) $y = 2 - x$

(d) $y = 0$

(e) $y = 2 + 2x$

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25.(6 pts.) Find the length of the polar curve between $\theta = 0$ and $\theta = 2\pi$

$$r = e^{-\theta}.$$

(a) $2e^{-4\pi}$

(b) $\frac{1}{4}(1 - e^{-4\pi})$

(c) $2 - e^{-2\pi}$

(d) $2\pi(1 + e^{-2\pi})$

(e) $\sqrt{2}(1 - e^{-2\pi})$

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The following is the list of useful trigonometric formulas:

$$\sin^2 x + \cos^2 x = 1$$

$$1 + \tan^2 x = \sec^2 x$$

$$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$$

$$\cos^2 x = \frac{1}{2}(1 + \cos 2x)$$

$$\sin 2x = 2 \sin x \cos x$$

$$\sin x \cos y = \frac{1}{2}\big(\sin(x - y) + \sin(x + y)\big)$$

$$\sin x \sin y = \frac{1}{2}\big(\cos(x - y) - \cos(x + y)\big)$$

$$\cos x \cos y = \frac{1}{2}\big(\cos(x - y) + \cos(x + y)\big)$$

The hyperbolic sine and cosine functions are defined to be:

$$\cosh x = \frac{e^x + e^{-x}}{2}$$

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

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