

Name: \_\_\_\_\_

Section: \_\_\_\_\_

**Math 10560, Quiz 10**

**April 18, 2023**

- The Honor Code is in effect for this quiz. All work is to be your own.
- Please turn off all cellphones and electronic devices.
- Calculators are NOT allowed
- The quiz lasts for 10 min.

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1. (a) (b) (c) (d) (e)

2. (a) (b) (c) (d) (e)

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### Multiple Choice

1.(2 pts.) Find the Maclaurin series for  $f(x) = 3^x$ .

- (a)  $\sum_{n=0}^{\infty} \frac{3^n}{n!} x^n$       (b)  $\sum_{n=0}^{\infty} \frac{(\ln 3)^n}{n!} x^n$       (c)  $\sum_{n=0}^{\infty} \frac{(-1)^n}{n!} x^{3n}$   
(d)  $\sum_{n=0}^{\infty} \frac{\ln 3}{n!} x^n$       (e)  $\sum_{n=0}^{\infty} \frac{x^n}{(3n)!}$

The Maclaurin series for  $f(x)$  is given by  $\sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n$ .

For  $f(x) = 3^x$ , the  $n$ th derivative is  $f^{(n)}(x) = 3^x (\ln 3)^n$ , so  $f^{(n)}(0) = (\ln 3)^n$ .

Plugging into the general formula above we see that the answer is **b**.

2.(2 pts.) Find the radius of convergence of the power series

$$\sum_{n=0}^{\infty} \frac{n+1}{(2n)!} (x-2)^n$$

- (a)  $R = 1$       (b)  $R = 0$       (c)  $R = \infty$   
(d)  $R = \frac{1}{2}$       (e)  $R = 2$

Applying the ratio test we have

$$\begin{aligned} \lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| &= \lim_{n \rightarrow \infty} \left| \frac{(n+2)(x-2)^{n+1}}{(2n+2)!} \cdot \frac{(2n)!}{(n+1)(x-2)^n} \right| \\ &= \lim_{n \rightarrow \infty} \left| \frac{(n+2)(x-2)(2n)!}{(n+1)(2n+2)(2n+1)(2n)!} \right| \\ &= |x-2| \lim_{n \rightarrow \infty} \frac{n+2}{(n+1)(2n+2)(2n+1)} = 0 \end{aligned}$$

So the series converges when  $0 < 1$ . This is always true, so the series converges for all  $x$ . Thus  $R = \infty$ , so the answer is **c**.

