Quiz 4 Solutions

1. Evaluate the definite integral:

$$\int_0^{\pi/4} \tan(\theta)\sec^6(\theta)d\theta.$$

Solution: The powers of $\tan(\theta)$ and $\sec(\theta)$ are such that we could either use $u = \tan(\theta)$ or $u = \sec(\theta)$. However, since we just have one copy of $\tan(\theta)$, the computation will be simpler with $u = \sec(\theta)$ and $du = \sec(\theta)\tan(\theta)d\theta$ since we won't need to use trig identities. So, we have

$$\begin{aligned}
\int_0^{\pi/4} \tan(\theta)\sec^6(\theta)d\theta &= \int_1^{\sqrt{2}} u^5 du \\
&= \left.\frac{u^6}{6}\right|_1^{\sqrt{2}} \\
&= \frac{2^3}{6} - \frac{1}{6} \\
&= \frac{7}{6}.
\end{aligned}$$

2. Evaluate the definite integral:

$$\int_0^{3/4} \frac{4}{9+16x^2}dx$$

Solution: We first factor out $\frac{1}{9}$, and then use u-substitution with $u = \frac{4x}{3}$ and $du = \frac{4dx}{3}$. We have

$$\begin{aligned}
\int_0^{3/4} \frac{4}{9+16x^2}dx &= \int_0^{3/4} \frac{1}{1+\left(\frac{4x}{3}\right)^2} \cdot \frac{4}{9}dx \\
&= \int_0^1 \frac{1}{1+u^2} \cdot \frac{1}{3}du \\
&= \frac{1}{3}\arctan(u)\Big|_0^1 \\
&= \frac{1}{3}\left(\frac{\pi}{4} - 0\right) \\
&= \frac{\pi}{12}.
\end{aligned}$$