Quiz 6 Solutions

1. Solve the equation:

$$y' = y\cos x$$

Solution:

We rewrite using Leibniz notation:

$$\frac{dy}{dx} = y\cos x$$

If $y \neq 0$ we can rewrite in differential notation and integrate:

$$\frac{dy}{y} = \cos x dx, \;\; y \neq 0$$
$$\int \frac{dy}{y} = \int \cos x dx$$
$$\ln|y| = \sin x + C$$

This defines y as an implicit function of x. We can solve explicitly for y as follows:

$$|y| = e^{\ln|y|} = e^{\sin x + C} = e^C e^{\sin x}$$

So

$$y = \pm e^C e^{\sin x}.$$

We can easily verify that $y = 0$ is a constant solution to the differential equation, so we can write the general solution in the form

$$y = Ae^{\sin x}$$

where A is a constant ($A = \pm e^C$ or $A = 0$).

Hence the correct answer is **a**.

2. Compute

$$\int_1^4 \frac{1}{x-3} dx.$$

Solution:

Since $\dfrac{1}{x-3}$ has a discontinuity at $x = 3$, we have to decompose the integral in two parts:

$$\int_1^4 \frac{1}{x-3} dx = \int_1^3 \frac{1}{x-3} dx + \int_3^4 \frac{1}{x-3} dx.$$

Now

$$\begin{aligned} \int_1^3 \frac{1}{x-3} dx &= \lim_{t\to 3^-} \int_1^t \frac{1}{x-3} dx \\ &= \lim_{t\to 3^-} \left(\ln|t-3| - \ln(1)\right) \\ &\to -\infty \end{aligned}$$

diverges, so the original integral diverges and the correct answer is **e**.