Quiz 7 Solutions

Q1: Solve the initial value problem:

$$\begin{aligned} \frac{dy}{dx} + 2xy &= x \\ y(0) &= -1. \end{aligned}$$

Solution: I.F. $= e^{\int 2x\,dx} = e^{x^2}$

$$\begin{aligned} e^{x^2}\frac{dy}{dx} + e^{x^2}2xy &= e^{x^2}x \\ \frac{d}{dx}(ye^{x^2}) &= xe^{x^2} \\ d(ye^{x^2}) &= xe^{x^2}dx \\ \int d(ye^{x^2}) &= \int xe^{x^2}dx \\ ye^{x^2} &= \frac{1}{2}e^{x^2} + C \end{aligned}$$

Now we can find the value of the integration constant by the given initial value.
As $y(0) = -1$, so

$$\begin{aligned} -1 &= \frac{1}{2} + C \\ C &= -\frac{3}{2} \end{aligned}$$

So we have,

$$\begin{aligned} ye^{x^2} &= \frac{1}{2}e^{x^2} - \frac{3}{2} \\ y &= \frac{1}{2} - \frac{3}{2}e^{-x^2} \end{aligned}$$

Hence the correct answer is **d**.

Q2: Consider the following sequence:

$$\left\{\frac{(-1)^n n}{2n+5}\right\}_{n=0}^{\infty}$$

Which one of the following statements is true?

Solution: An alternating sequence $\{(-1)^n a_n\}$ converges iff $lim_{n\to\infty} a_n = 0$.
Here, $a_n = \frac{n}{2n+5}$
As $lim_{n\to\infty} a_n = \frac{1}{2}$, so our sequence does not converge.

Hence the correct answer is **a**.