

Name: _____

Section: _____

Math 10560, Quiz 8

March 28, 2023

- The Honor Code is in effect for this quiz. All work is to be your own.
- Please turn off all cellphones and electronic devices.
- Calculators are NOT allowed
- The quiz lasts for 10 min.

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1. (a) (b) (c) (d) (e)

2. (a) (b) (c) (d) (e)

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Multiple Choice

1.(2 pts.) Which one of the following series is convergent?

(a) $\sum_{n=1}^{\infty} \frac{n}{n^2 + 2}$

(b) $\sum_{n=1}^{\infty} \frac{1}{n^{1/2}}$

(c) $\sum_{n=1}^{\infty} \frac{3^n}{n^3}$

(d) $\sum_{n=1}^{\infty} \frac{3^{-n}}{n^3}$

(e) $\sum_{n=1}^{\infty} \frac{1}{n}$

For (a), $\frac{n}{n^2 + 2} \geq \frac{n}{2 \cdot n^2} = \frac{1}{2n}$ if $n \geq 2$, by the comparison test, it diverges. (b) and (e) clearly diverge by the p -series test. For (c), one can use L'Hopital's rule to show $\lim_{n \rightarrow \infty} \frac{3^n}{n^3} = \infty$.

Finally, $\frac{3^{-n}}{n^3} \leq \frac{1}{n^3}$, so (d) converges by the comparison test. Hence, the answer is (d).

2.(2 pts.) Consider the improper integral

$$\int_{-1}^1 \frac{1}{3x - 1} dx.$$

which of the following statements is true?

(a) It converges to 0. (b) It converges to $\frac{1}{3}$. (c) It converges to $\frac{1}{2} \ln \frac{1}{2}$.

(d) It converges to $\ln \frac{1}{2}$. (e) It diverges.

The answer is (e): Notice that $x = 1/3$ is a singular point (problem point), so we need to break the integral in to 2 parts

$$\int_{-1}^1 \frac{1}{3x - 1} dx = \int_{-1}^{1/3} \frac{1}{3x - 1} dx + \int_{1/3}^1 \frac{1}{3x - 1} dx.$$

In this case, the integral on the left hand side converges if both of the integral on the right hand side converge. However, it's easy to see both integral on the right diverge.

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