Quiz 9 Solutions

1. Using the limit comparison test, determine whether the following series converge or diverge:

$$(i)\sum_{n=1}^{\infty}\frac{1}{n^{1/3}+2}, \qquad (ii)\sum_{n=1}^{\infty}\frac{10}{9+7^n}, \qquad (iii)\sum_{n=1}^{\infty}\frac{2n^3-4n+10}{n^7+8}.$$

Solution:

For (i), compare to $\sum \frac{1}{n^{1/3}}$. When we take the limit of the quotient, we get $\lim_{n\to\infty}\frac{n^{1/3}}{n^{1/3}+2}=1>0$. Thus, since we're comparing to a divergent series, this series diverges as well.

For (ii), compare to $\sum \frac{1}{7^n}$. We get $\lim_{n\to\infty}\frac{10}{9+7^n}\cdot\frac{7^n}{1}=10\lim_{n\to\infty}\frac{7^n}{9+7^n}=10>0$. Since we're comparing to a convergent series, this series converges as well.

For (iii), compare to $\sum \frac{1}{n^4}$. We get $\lim_{n\to\infty}\frac{2n^3-4n+10}{n^7+8}\cdot\frac{n^4}{1}=\lim_{n\to\infty}\frac{2n^7-4n^5+10n^4}{n^7+8}=2>0$. Since we're comparing to a convergent series, this series converges as well.

So, the correct answer is (e).

2. What can be said about the following series using the Root Test?

$$(i)\sum_{n=1}^{\infty}\left(\frac{5-8n^3}{7+8n^3}\right)^n, \qquad (ii)\sum_{n=1}^{\infty}\left(\frac{5n}{n+1}\right)^n, \qquad (iii)\sum_{n=1}^{\infty}\left(\frac{1}{n!}\right)^n$$

Solution:

For (i), $\lim_{n\to\infty}\sqrt[n]{\left|\left(\frac{5-8n^3}{7+8n^3}\right)^n\right|}=\lim_{n\to\infty}\frac{8n^3-5}{7+8n^3}=1$, so the Root Test is inconclusive.

For (ii), $\lim_{n\to\infty}\sqrt[n]{\left|\left(\frac{5n}{n+1}\right)^n\right|}=\lim_{n\to\infty}\frac{5n}{n+1}=5>1$. Thus, the series diverges by the Root Test.

For (iii), $\lim_{n\to\infty}\sqrt[n]{\left|\left(\frac{1}{n!}\right)^n\right|}=\lim_{n\to\infty}\frac{1}{n!}=0<1$. Thus, the series converges by the Root Test.

So, the correct answer is (a).