Fundamentals of Fan Aeroacoustics
Overview of Lecture

• **Noise Sources and Generation Mechanisms**
  – **Sources of Noise for Typical Fans**
  – **Fluid-Structure Interaction as a Noise Generation Mechanism**
  – **Coupling to the Duct: Propagating Modes and Cut-off Phenomena**

• **Modeling of Fan Noise**
  
  **The Acoustic Analogy**
  **Computational Methods:** *Aeroacoustics and Unsteady Aerodynamics*
  **The Linear Cascade Model**
  **Effects of Geometry and Blade Loading on Acoustic Radiation**

• **Recent Developments in Fan Modeling**
  – **Tonal and Broadband Noise**
  – **Nonuniform Mean Flow Effects:** *swirl*
  – **Three-Dimensional Effects**
  – **High Frequency Effects**

• **Conclusions**
Dominant Noise Sources for Turbofan Engines

- Inflow disturbances
- Inlet boundary layer
- Strut potential field
- Compressor inlet
- Rotor
- Wakes
- Vortices
- Turbulence
- Rotor leading edge shocks
- Turbine exit
- Acoustic treatment
- Stator
High Bypass Ratio Fan Flyover Noise
Maximum Perceived Noise Level

Approach

Takeoff

Hard wall
(no acoustic
treatment)

With
acoustic
treatment
Typical Turbomachinery Sound Power Spectra

Subsonic Tip Speed

Supersonic Tip Speed
Turbomachinery Noise Generation Process

1. Internal disturbances (blade wakes, vortices, turbulence)
2. Inflow disturbances (turbulence, vortices, wakes)
3. Blade response
4. Unsteady blade surface pressures
5. Acoustic coupling to duct
6. Duct acoustic mode content
7. Duct propagation blade row transmission
8. Duct exit acoustic mode content
9. Acoustic coupling to far field
10. Far-field directivity

Flow characteristics:
- Supersonic flow over blades
- Rotating shock structure

Suppressor properties (impedance or propagation characteristics)

SPL vs. θ
Rotor-Stator Interaction
Fluid-Structure Interaction as a Noise Generation Mechanism

- The interaction of nonuniform flows with structural components such as blades and guide vanes produce fluctuating aerodynamic forces on the blades and radiates sound in the farfield.

- Noise Sources: Flow Nonuniformities: Inlet Turbulence, Boundary Layers, Tip and Hub Vortices, Wakes etc.


- Propagation in the Duct: Sound Must Propagate in a Duct: therefore only high frequency acoustic modes will propagate.
Rotor Wakes Interaction with Downstream Stator

- Rotor
- Stator
- Wake
- Wakes
- Source of unsteady flow

Diagram showing the interaction between rotor wakes and downstream stator, with vectors indicating flow directions and angles.
Rotor-Stator Interaction
Wakes and Tip Vortices
Scaling Analysis

• Multiple length scales:
  – Duct hub and tip Radii: $R_h$, $R_t$, Rotor/stator spacing: $L$
  – Chord length $c$, Blade spacing $s=2pR/(B$ or $V$)
  – Turbulence Integral Scale $= \Lambda << c << R$

• Multiple Frequency Parameters:

\[
\frac{\omega R}{c_o} = \frac{B \Omega R}{c_o} = BM \theta >> 1 \quad \frac{\omega \Lambda}{U_x} = \left( \frac{B \Lambda}{R} \right) \left( \frac{U^s}{U_x} \right) = O(1)
\]

• Fast Variables: \( \tilde{X} = \frac{\overset{\sim}{X}}{\Lambda} \)  
  Blade passing frequency $\text{BPF}=B\Omega$

• Slow Variables: \( \bar{X}^* = \frac{\overset{-}{X}}{R} \)
Multiple Pure Tones

(a) Idealized wave pattern.

(b) Actual wave pattern.
Airfoil in Nonuniform Flow

- Unsteady Nonuniform Upstream Flow
- Unsteady Pressure in the Near Field
- Acoustic Radiation in the Far Field
- Vortex Shedding in the Wake
Equations of Classical Acoustics

\[ p = p_0 + p' \]
\[ \rho = \rho_0 + \rho' \]
\[ \left| \vec{u} \right| \ll c_0 \]
\[ \frac{\partial \rho'}{\partial t} + \rho_0 \nabla \cdot \vec{u} = 0 \]
\[ \frac{\partial \vec{u}}{\partial t} + \frac{1}{\rho_0} \nabla p' = 0 \]
\[ \left( \nabla^2 - \frac{1}{c_0^2} \frac{\partial^2}{\partial t^2} \right) p' = 0 \]
Acoustic Intensity and Energy

• Fundamental Conservation Equation

\[ \frac{\partial E}{\partial t} + \nabla \cdot \vec{I} = 0 \]

\[ E = \frac{p' \rho'}{2 \rho_0} + \frac{\rho_0 u^2}{2} \]

\[ \vec{I} = p' \vec{u} \]
Fundamental Solution of the wave Equation

- **Spherical symmetry:**
  \[ f(t \mp \frac{R}{c}) \]
  \[ \Phi = \frac{\delta(t - \tau - \frac{R}{c})}{c}, \quad R = |\vec{x} - \vec{y}| \]

- **Green’s Function**
  \[ G(\vec{x}, t \mid \vec{y}, \tau) = \frac{\delta(t - \tau - \frac{R}{c})}{c} \]
  t-R/c: retarded time

- **Compact/Noncompact Sources**

Causality determines the sign: Sound must propagate away from the source
Plane Waves

\[ p' = \overline{pe} i(\vec{k} \cdot \vec{x} - \omega t) \]

\[ \ddot{u} = \frac{\overline{p}}{\rho_0 c_0} \frac{\vec{k}}{k} e^{i(\vec{k} \cdot \vec{x} - \omega t)} = \frac{p'}{\rho_0 c} \frac{\vec{k}}{k} \]

\[ E = \frac{p' \rho'}{2 \rho_0} + \frac{\rho_0 u^2}{2} = \frac{p'^2}{\rho_0 c_0} \]

\[ \vec{I} = p' \ddot{u} = \frac{p'^2}{\rho_0 c_0} \frac{\vec{k}}{k} \]

\[ \vec{I} = \frac{1}{2} \frac{\overline{p}^2}{\rho_0 c_0} \]
## Elementary Solutions of the Wave Equation

### No Flow

<table>
<thead>
<tr>
<th>Noise Source</th>
<th>Acoustic Pressure</th>
<th>Acoustic Intensity</th>
<th>Directivity</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Point Source</strong></td>
<td>( \ddot{m} \left( t - \frac{R}{c} \right) ) ( \frac{4 \pi R}{4 \pi R} )</td>
<td>( \ddot{m} \left( t - \frac{R}{c} \right) ) ( \frac{16 \pi^2 \rho_0 R^2 c}{16 \pi^2 \rho_0 R^2 c} )</td>
<td>Spherical symmetry</td>
</tr>
<tr>
<td><strong>Dipole: Force</strong></td>
<td>( \ddot{F}(t - \frac{R}{c}).\vec{e}_r ) ( \frac{4 \pi R c}{4 \pi R c} )</td>
<td>( \frac{\pi}{4} )</td>
<td>( \cos \theta )</td>
</tr>
<tr>
<td><strong>Quadrupole: Stress</strong></td>
<td>( \dddot{T}_{ij} V_x x_i x_j ) ( \frac{4 \pi R^3 c^2}{4 \pi R^3 c^2} )</td>
<td>( \dddot{T}<em>{ij} T</em>{kl} V^2 x_i x_j x_k x_l ) ( \frac{16 \pi^2 \rho R^6 c^4}{16 \pi^2 \rho R^6 c^4} )</td>
<td>( \cos \theta \sin \theta )</td>
</tr>
</tbody>
</table>
Directivity of Elementary Sources of sound

Monopole Directivity

Dipole Directivity

Quadrupole Directivity
Scaling of Acoustic Power Radiated for Aerodynamic Applications at Low Mach Number

<table>
<thead>
<tr>
<th>Source</th>
<th>Scaling</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dipole</td>
<td>$M^6$</td>
<td>1</td>
</tr>
<tr>
<td>Quadrupole</td>
<td>$M^8$</td>
<td>$M^2$</td>
</tr>
</tbody>
</table>

Thus at low Mach number dipoles or forces are more efficient sources of noise than quadrupoles or turbulence. However, this result is valid only for uniform flows and low frequency.
As the vortex travels near the trailing edge it is no longer convected by the mean flow. Its trajectory crosses the undisturbed mean flow. This increases the amount of fluid energy converted into acoustic energy. The acoustic power scales with $M^3$, much higher than that predicted by a dipole ($M^6$).
Acoustic Waves in Ducts

Square Duct

- Higher order modes

\[ p' = \cos \frac{m \pi x_2}{a} \cos \frac{m \pi x_3}{a} \left[ A_{mn} e^{i(k_{mn}x_1 - \omega t)} + B_{mn} e^{-i(k_{mn}x_1 + \omega t)} \right] \]

- Dispersion Relation

\[ k_{mn} = \sqrt{\frac{\omega^2}{c^2} - \frac{\pi^2}{a^2} (m^2 + n^2)} \]

- Propagating or Cut-on Modes
  - \( k_{mn} \): real
    - Evanescent Modes
  - \( k_{mn} \): imaginary

Plane waves always propagate.
Higher order modes propagate only when

\[ \omega > \frac{\pi c}{a} \quad \text{or} \quad \lambda < 2a \]

The velocity and pressure of evanescent or cut-off modes are out of phase and there is no net transport of energy (I=0).
Phase and Group Velocities for Dispersive Waves

- **Phase Velocity**
  \[ c_{ph} = \frac{\omega}{k} = \frac{c}{\sqrt{1 - \pi^2 (m^2 + n^2) \frac{c^2}{a^2 \omega^2}}} \]
  - Phase velocity is larger than the speed of sound

- **Group Velocity**
  \[ c_g = \frac{d\omega}{dk} = c \sqrt{1 - \pi^2 (m^2 + n^2) \frac{c^2}{a^2 \omega^2}} \]
  - Group velocity is the velocity at which acoustic energy is transported in the duct. It is smaller than the speed of sound.
• Reflecting wave components making up the first higher mode propagating between two parallel plates. Solid lines represent pressure maxima of the wave; dotted lines, pressure minima. Arrows, representing direction of propagation of the components, are normal to the wavefronts.
Application to Rotor/Stator Interaction

The Tylor-Sofrin Modes

- Incident Gust
  \[ \vec{u}_g = \tilde{a}_m^{(g)}(r)e^{i(k_g x + m'\theta - \omega t)}, \quad k_g = \frac{\omega}{U} \]
  \( \sigma = m' \vartheta = \frac{2\pi}{V} \)

- Cut-on Modes
  \[ \vec{u}_a = \sum_{m,n} \tilde{a}_{m,n}(r)e^{i(k_{mn} x + m\theta - \omega t)} \]
  \( m = m' - qV \)

- For rotor/stator interaction with B rotor blades and V guide vanes
  \( m' = pB \)

- Hence
  \( m = pB - kV \)

Example: B=18, V=40. m=-22, -4, 10, 14, 22
Sound Propagation in a Duct with Uniform Flow

- The governing equation
\[
\frac{D_0^2 p'}{D_t^2} + \nabla^2 p' = 0
\]

- With the boundary condition at the hub and the tip
\[
\frac{\partial p'}{\partial r} = 0
\]

- The eigenvalues are given by,
\[
k_{mn} = -\tilde{\omega}M_x \pm \sqrt{\tilde{\omega}^2 - (1 - M_x^2)(m^2 + \gamma_{mn}^2)} - \frac{1}{1 - M_x^2}
\]

Where
\[
\tilde{\omega} = \omega - m M_s
\]

- The solution is given by
\[
p' = \sum_{m=-\infty}^{\infty} \sum_{n=0}^{\infty} K_n(\gamma_{mn} r) e^{i(k_{mn} x + m \theta - \omega t)}
\]
Conclusions

• Fan noise sources: Flow nonuniformities and irregular flow pattern.
• Mechanism: Fluid/structure interaction and link to unsteady aerodynamics.
• Classical acoustics concepts are essential to understanding and modeling of noise.
• The coupling to the duct determines the modal content of the scattered sound and affects sound propagation.
Modeling of Fan Noise

- The Acoustic Analogy
- Computational Methods: Aeroacoustics and Unsteady Aerodynamics
- The Linear Cascade Model
- Effects of Geometry and Blade Loading on Acoustic Radiation
Lighthill’s Acoustic Analogy

- Inhomogeneous wave equation
  \[
  \frac{\partial^2 \rho}{\partial t^2} - c_0^2 \nabla^2 \rho = \frac{\partial^2 T_{ij}}{\partial x_i \partial x_j}
  \]

- Lighthill’s stress tensor
  \[
  T_{ij} = \rho v_i v_j + \delta_{ij} (p - c_0^2 \rho) - e_{ij}
  \]
  \[
  e_{ij} = \mu \left( \frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right) - \frac{2}{3} \mu \delta_{ij} \frac{\partial v_k}{\partial x_k}
  \]

- For a Uniform Mean Flow
  \[
  \frac{D_0^2 \rho}{\partial t^2} - c_0^2 \nabla^2 \rho = \frac{\partial^2 T_{ij}}{\partial x_i \partial x_j}
  \]
Aeroacoustics and Unsteady Aerodynamics

- Sound is the far-field signature of the unsteady flow.
- Unsteady aerodynamics has been developed for aeroelastic problems such as flutter and forced vibrations where the main interest is to determine the near-field body surface forces.
- The aeroacoustic problem is similar to that of forced vibration but with emphasis on the far-field. It is a much more difficult computational problem whose outcome depends on preserving the far-field wave form with minimum dispersion and dissipation.
- Inflow/outflow nonreflecting boundary conditions must be derived to complete the mathematical formulation as a substitute for physical causality.
Airfoil in Nonuniform Flow

Unsteady Nonuniform Upstream Flow

Unsteady Pressure in the Near Field

Vortex Shedding in the Wake

Acoustic Radiation in the Far Field
Disturbances in Uniform Flows

Splitting Theorem:

The flow disturbances can be split into distinct potential (acoustic), vortical and entropic modes obeying three independent equations.

- The vortical velocity is solenoidal, purely convected and completely decoupled from the pressure fluctuations.
- The potential (acoustic) velocity is directly related to the pressure fluctuations.
- The entropy is purely convected and only affects the density through the equation of state.
- Coupling between the vortical and potential velocity occurs only along the body surface.
- Upstream conditions can be specified independently for various disturbances.
Splitting of the Velocity into Acoustic, Entropic and Vortical Modes

\[ \vec{V} (\vec{x}, t) = \vec{U} + \vec{u} (\vec{x}, t) \]

\[ \frac{D}{Dt} \rho' + \rho_0 \nabla \cdot \vec{u} = 0 \]

\[ \frac{D}{Dt} \vec{u} + \rho_0 \nabla \cdot p' = 0 \]

\[ \frac{D}{Dt} \equiv \frac{\partial}{\partial t} + \vec{U} \cdot \nabla \]

\[ p' = -\rho_0 \frac{D}{Dt} \phi \]

\[ \vec{u} = \vec{u}_v + \nabla \phi \]

\[ \frac{D}{Dt} \vec{u}_v = 0 \]

\[ \left( \frac{1}{c_0^2} \frac{D}{Dt}^2 - \nabla^2 \right) \phi = 0 \]

\[ \frac{D}{Dt} s' = 0 \]
Equations for Linear Aerodynamics

• Vortical Mode:
  • Harmonic Component

\[
\vec{u}_v = \vec{u}_\infty (\vec{x} - \vec{U}i)
\]

\[
\vec{u}_g = \tilde{a}e^{i(\vec{k} \cdot \vec{x} - \omega t)}
\]

• Potential Mode:

\[
\left( \frac{1}{c_0^2} \frac{D}{Dt} \frac{2}{2} - \nabla^2 \right) \phi = 0
\]

• Boundary Conditions: impermeability along blade surface, Kutta condition at trailing edge, allow for wake shedding in response to gust.
Flat Plate in a Gust

\[ \vec{u} = \vec{a} e^{i(\vec{k} \cdot \vec{x} - \omega t)} \]

Transverse Gust: \((0, a_2, 0), (k_1, 0, 0)\)

Oblique Gust: \((0, a_2, 0), (k_1, 0, k_3)\)
Vector Diagram Showing the Real and Imaginary Parts of the Response Function $S(k_1,0,M)$ versus $k_1$ for a Transverse Gust at Various $M$
Vector Diagram Showing the Real and Imaginary Parts of the Response Function $S(k_1,k_3,0.8)$ versus $k_1$ for a Transverse Gust at Various $M$
Airfoil in Three-Dimensional Gust
VORTEX STRETCHING AT THE STAGNATION POINT
Vector Diagram Showing the Real and Imaginary Parts of the Response Function $S(k_1,k_2,k_3)$ versus $k_1$ for an Airfoil in an Oblique Gust ($k_2=k_1$) at Various $k_3$. 

A1=$-0.71$
A2=$0.71$
K2=$K_1$
A3=$0$
INCIDENCE=$10$ DEG

THE UNSTEADY LIFT COEFFICIENT
JOUKOWSKI AIRFOIL: CAMBER=$0.1$ THICKNESS=$0.1$
Acoustic Directivity for a 3% Thick Airfoil in a Transverse Gust at \( M=0.1, k_1=1.0 \)

- - - , direct calculation from Scott-Atassi’s code; —, Kirchhoff’s method; ---, flat plate semi-analytical results.
Acoustic Pressure Directivity for a Symmetric Airfoil in a Two-Dimensional Gust

Thickness Ratio = 0.06, M = 0.7, $k_1 = 3.0$

$k_2$: solid line, 0.0; ------, 3.0.

(b) Dipole Acoustic Pressure

(a) Total Acoustic Pressure
Acoustic Pressure Directivity for a Lifting Airfoil in a transverse Gust

Thickness Ratio=0.12, M=0.5, k₁=2.5
Camber: solid line,0.0; ------,0.2; -- . --, 0.4.
Conclusions

• At low Mach number, low frequency, dipole effects (unsteady airfoil pressure) dominate the scattered sound.
• At moderate and high Mach number and/or high frequency, the scattered sound strongly depends on both dipole and quadrupole effects and sound directivity is characterized by lobe formation.
• Loading strongly affects the scattered acoustic energy.
• Exact nonreflecting boundary conditions are essential for obtaining accurate results particularly at high Mach number and reduced frequency.
The Linear Cascade Model

- Separate rotor and stator and consider each blade row separately.
- Unroll the annular cascade into a linear cascade of infinite blade to preserve periodicity.
- **Flat-plate cascade**: uniform mean flow, integral equation formulation in terms of plane waves. The current benchmark.
- **Loaded cascade**: Linearized Euler about a computationally calculated mean flow, requires field solutions of pde. CASGUST and LINFLOW are current benchmarks.
Unrolling of the Annular Cascade
The Linear Cascade
Linear Cascade and Strip Theory

FLOW IN CASCADE
Cascade of Airfoils in a Three-Dimensional Gust
Figure 3: Real and imaginary parts of the unsteady lift versus the reduced frequency $k_1$ for a cascade of flat plates ($\chi = 45^\circ$, $\frac{S}{c} = 1.0$, $M_\infty = 0.3$, $k_2 = 0$)
Figure 5: Real and imaginary parts of the unsteady lift versus the reduced frequency $k_1$ for a cascade of flat plates ($\chi = 45^\circ$, $\frac{S}{c} = 1.0$, $M_\infty = 0.3$, $k_2 = k_1$)

Figure 6: The first four modes of the sound radiated from a cascade of flat plates ($\chi = 45^\circ$, $\frac{S}{c} = 1.0$, $M_\infty = 0.3$, $k_2 = k_1$)
Figure 11: Real and imaginary parts of the unsteady lift versus the reduced frequency $k_1$ for a cascade of NACA 4406 blades ($\chi = 35^\circ$, $\frac{S}{c} = 1.0$, $M_\infty = 0.3$, $k_2 = 0$)

Figure 12: The first four modes of the sound radiated from a cascade of NACA 4406 blades ($\chi = 35^\circ$, $\frac{S}{c} = 1.0$, $M_\infty = 0.3$, $k_2 = 0$)
Figure 13: Real and imaginary parts of the unsteady lift versus the reduced frequency $k_1$ for a cascade of NACA 4406 blades ($\chi = 35^\circ$, $\frac{S}{c} = 1.0$, $M_\infty = 0.3$, $k_2 = k_1$)

Figure 14: The first four modes of the sound radiated from a cascade of NACA 4406 blades ($\chi = 35^\circ$, $\frac{S}{c} = 1.0$, $M_\infty = 0.3$, $k_2 = k_1$)
Magnitude of the Response Function $S$ versus $k_1$ for an EGV Cascade (squares) with Flat-Plate Cascade (circles). $M=0.3$, $k_2=k_1$
Magnitude of the Downstream Acoustic Modes from an EGV Cascade with Those of a Flat Plate Cascade (solid)

\[ M=0.3, \quad k_2=k_1 \]
How Good is the Linearized Euler Model?
Cascade Flow with local Regions of Strong Interaction

- Leading edge separation
- Shock/boundary-layer interaction
- Trailing-edge separation
- Strong-interaction regions
- Weak interactions
- $V_\infty$
COMPRESSOR EXIT GUIDE VANE

NGUST Computational Grid
HIGH SPEED COMPRESSOR CASCADE

Surface Mach Number Distributions

--- Potential, - - - - Euler

Subsonic flow

Transonic flow

M

Suction surface

Pressure surface

Suction surface

Pressure surface
Magnitude (a) and Phase (b) of the first Harmonic Unsteady Pressure Difference Distribution for the subsonic NACA 0006 Cascade Underhoing an In-Phase Torsional Oscillation of Amplitude $a=2\phi$ at $k1=0.5$ about Midchord; $M=0.7$; —, Linearized Analysis; .... Nonlinear Analysis.
Magnitude (a) and Phase (b) of the first Harmonic Unsteady Pressure Difference Distribution for the subsonic NACA 0006 Cascade Undergoing an In-Phase Torsional Oscillation of Amplitude $a=2\omega$ at $k_1=0.5$ about Midchord; $M=0.7$. 
Conclusions

- Cascade effects such as blade interference which depends on the spacing ratio, and stagger have strong influence on the aerodynamic and acoustic cascade response, particularly at low frequency.
- At high frequency the cascade response is dominated by the acoustic modes cut-on phenomena.
- For thin blades, the leading edge dominates the aerodynamic pressure and noise generation.
- For loaded blades at high Mach number, large unsteady pressure excitations spread along the blade surface with concentration near the zone of transonic velocity.
- Blade loading changes the upstream and downstream flows and thus affects the number and intensity of the scattered sound.
Recent Developments in Fan Modeling

- **Tonal and Broadband Noise**
- **Nonuniform Mean Flow Effects**: swirl
- **Three-Dimensional Effects**
- **High Frequency Effects**
Tonal and Broadband

• Turbulence modeling using the rapid distortion theory.
• Hanson (Pratt & Whitney), Glegg (Florida) developed models using linear flat-plate cascades.
• Effects of blade loading, 3D effects are under development at ND
Turbulence Representation

- Fourier representation:
  \[
  \vec{u}(\vec{x}, t) = \int_{\omega, \vec{k}} a(\vec{k}, \omega) e^{i(\vec{k} \cdot \vec{x} - \omega t)} \, d\vec{k} d\omega
  \]
  \[
  a_i(\vec{k}) a_j(\vec{k}') = \Phi_{ij}(\vec{k}) \delta(\vec{k} - \vec{k}')
  \]

- Evolution of each Fourier component
  \[
  u_i(\vec{x}, t) = \mu_{ij}(\vec{x}, \vec{k}) a_j e^{-ik_l Ut}
  \]

- Velocity covariance
  \[
  R_{ij}(\vec{x}, \vec{x}', t) = \int_{\vec{k}} \mu^*_{ik}(\vec{x}, \vec{k}) \mu_{j\ell}(\vec{x}', \vec{k}) \Phi^{(0)}_{k\ell}(\vec{k}) e^{-ik_l Ut} \, d\vec{k}
  \]

- One-dimensional energy spectrum
  \[
  \Theta_{ij}(\vec{x}, \vec{x}', k_1) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \mu^*_{ik}(\vec{x}, \vec{k}) \mu_{j\ell}(\vec{x}', \vec{k}) \Phi^{(0)}_{k\ell}(\vec{k}) \, dk_2 \, dk_3
  \]
Aerodynamic and Acoustic Blade Response

- Swirling Mean flow + disturbance
- Rapid distortion theory
  "disturbance propagation"
- Normal mode analysis
  "construction of nonreflecting boundary conditions"
- Source term on blades
- Blade unsteady loading & radiated sound field
- Non-reflecting boundary conditions

Linearized Euler model
Mathematical Formulation

• Linearized Euler equations
• Axisymmetric swirling mean flow
\[ \tilde{U}(\tilde{x}) = U_x(x, r)\tilde{e}_x + U_s(x, r)\tilde{e}_s \]
• Mean flow is obtained from data or computation
• For analysis the swirl velocity is taken
\[ U_s = \Omega r + \frac{\Gamma}{r} \]
• The stagnation enthalpy, entropy, velocity and vorticity are related by Crocco’s equation
\[ \nabla H = TVS + U \times \zeta \]
Normal Mode Analysis

- A normal mode analysis of linearized Euler equations is carried out assuming solutions of the form

\[ f(r)e^{i(-\omega t + m\nu \theta + k_{mn} x)} \]

- A combination of spectral and shooting methods is used in solving this problem
  - Spectral method produces spurious acoustic modes
  - Shooting method is used to eliminate the spurious modes and to increase the accuracy of the acoustic modes

\[ \Lambda_{mn} = \frac{D_0}{Dt} = -\omega + k_{mn} U_x + \frac{mU_s}{r} \]
Mode Spectrum
Spectral and Shooting Methods

\[ M_x = 0.55, \quad M_\Gamma = 0.24, \quad M_\Omega = 0.21, \quad \omega = 16, \quad \text{and} \quad m = -1 \]
Pressure Content of Acoustic and Vortical Modes

\[ M_x=0.5, \ M_\Gamma=0.2, \ M_\Omega=0.2, \ \omega=2\pi, \ \text{and} \ m=-1 \]
Effect of Swirl on Eigenmode Distribution

\( M_{xm}=0.56, M_\Gamma=0.25, M_\Omega=0.21 \)
Summary of Normal Mode Analysis

Normal Modes

Pressure-Dominated Acoustic Modes
- Propagating
- Decaying
- Nonreflecting Boundary Conditions

Vorticity-Dominated Nearly-Convected Modes
- Singular Behavior

Nonreflecting Boundary Conditions
Nonreflecting Boundary Conditions

- Accurate nonreflecting boundary conditions are necessary for computational aeroacoustics.
Formulation

- Pressure at the boundaries is expanded in terms of the acoustic eigenmodes.

\[
p(\vec{x}, t) = \int \sum_{\omega} \sum_{n=0}^{\infty} c_{mn} p_{mn}(\omega, r) e^{i(-\omega t + m\theta + k_{mn} x)} d\omega
\]

- Only outgoing modes are used in the expansion.
- Group velocity is used to determine outgoing modes.
Nonreflecting Boundary Conditions (Cont.)

\[
p(\vec{x}, t) = \sum_{\nu = -M/2}^{M/2} \sum_{n = 0}^{N} c_{mn} p_{mn}(r) e^{i(\omega t + m_r \nu + k_{mn} x)}
\]

\[
[p] = [\mathcal{R}] c
\]

Computational Domain

\[
[p]_{L-1} = [\mathcal{R}]_{L-1} [c]
\]

\[
[p]_L = [\mathcal{R}]_L [c]
\]

\[
[p]_L = [\mathcal{R}]_L [\mathcal{R}]_{L-1}^{-1} [p]_{L-1}
\]
The rotor/stator system is decoupled

The upstream disturbance can be written in the form,

\[
\bar{u}_1(r, \theta) = \sum_{m' = -\infty}^{\infty} \bar{a}_m(r)e^{i(m'\theta - \omega t)}
\]

\[
p_1(r, \theta) = \sum_{m' = -\infty}^{\infty} A_m(r)e^{i(m'\theta - \omega t)}
\]

- Quasi-periodic conditions
- Wake discontinuity
- Nonreflecting conditions
Numerical Formulation

- The rotor/stator system is decoupled
- The upstream disturbance can be written in the form,

\[
\tilde{u}_1(r, \theta) = \sum_{m'=-\infty}^{\infty} \tilde{a}_{m'}(r)e^{i(m'\theta - \omega t)}
\]

\[
p_1(r, \theta) = \sum_{m'=-\infty}^{\infty} A_{m'}(r)e^{i(m'\theta - \omega t)}
\]

- Quasi-periodic conditions
- Wake discontinuity
- Nonreflecting conditions

Two Vanes

Tip

Hub

Computational Domain
Domain Decomposition
Inner and Outer Regions

Vorticity and Pressure are Coupled.
Solution is Simplified by Removing Phase.

Vorticity and Pressure Are uncoupled.

O(R)          O(c)
Scattering Results
## Parameters for Swirling Flow Test Problem

<table>
<thead>
<tr>
<th>Narrow Annulus</th>
<th>Full Annulus</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_{\text{tip}}/r_{\text{hub}}$</td>
<td>1.0/0.98</td>
<td>1.0/0.75</td>
</tr>
<tr>
<td>$\omega$</td>
<td>0.5$\pi$, 1.0$\pi$, 1.5$\pi$, 2.0$\pi$, 2.5$\pi$, 3.0$\pi$, 3.5$\pi$, 4.0$\pi$</td>
<td>$\omega$</td>
</tr>
<tr>
<td>$M$ (mach number)</td>
<td>0.5</td>
<td>$\alpha$ (disturbance)</td>
</tr>
<tr>
<td>$B$ (rotor blades)</td>
<td>16</td>
<td>$V$ (stator blades)</td>
</tr>
<tr>
<td>$C$ (chord)</td>
<td>$2\pi/V$</td>
<td>Stagger</td>
</tr>
<tr>
<td>$L$ (length)</td>
<td>3$c$</td>
<td></td>
</tr>
</tbody>
</table>
## Computational Domain

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>B</strong> (rotor blades)</td>
<td>16</td>
</tr>
<tr>
<td><strong>V</strong> (stator blades)</td>
<td>24</td>
</tr>
<tr>
<td><strong>C</strong> (chord)</td>
<td>(2\pi/\sqrt{V})</td>
</tr>
<tr>
<td><strong>L</strong> (length)</td>
<td>3c</td>
</tr>
</tbody>
</table>

\[
\omega = \frac{nB\Omega r_m}{c_o} \approx \mathcal{O}(nB)
\]
Narrow Annulus Limit

<table>
<thead>
<tr>
<th>$r_{\text{tip}}/r_{\text{hub}}$</th>
<th>1.0/0.98</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega$</td>
<td>$1.5\pi, 4.5\pi, 6.5\pi$</td>
</tr>
<tr>
<td>$M$</td>
<td>0.5</td>
</tr>
<tr>
<td>Stagger</td>
<td>45</td>
</tr>
</tbody>
</table>
Narrow Annulus Limit
Gust Response for Narrow Annulus Limit –Comparison with 2D cascade

\[ M_o = 0.3536, \ M_\theta = 0.3536, \ B = 16, \ V = 24, \ a_r = 0 \]

**Unsteady Lift Coefficient**

**Acoustic Coefficients**

\[ m = -8, \ n = 0, 1 \]
Gust Response Effect of Hub-Tip Ratio

\[ M_o = 0.3536, \; M_I = 0.1, \; M_\Omega = 0.1, \; \omega = 3\pi, \; \text{and} \; a_r = 0 \]

Unsteady Lift Coefficient

Acoustic Coefficients

\[ m = -8, \; n = 0, 1 \]
Mean Flow Cases

Mean Flow 1: $M_o=0.4062$, $M_\Gamma=0.0$, $M_\Omega=0.0$

Mean Flow 2: $M_o=0.3536$, $M_\Gamma=0.2$, $M_\Omega=0.1$

Mean Flow 3: $M_o=0.3536$, $M_\Gamma=0.0$, $M_\Omega=0.2$

Mean Flow 4: $M_o=0.3536$, $M_\Gamma=0.1$, $M_\Omega=0.1$
Effect of Mean Flow

\[ r_h/r_t=0.6, \ \omega=3\pi, \ \text{and} \ a_r=0 \]

<table>
<thead>
<tr>
<th>Mode m=−8</th>
<th>Mean Flow 1</th>
<th>Mean Flow 2</th>
<th>Mean Flow 3</th>
<th>Mean Flow 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Downstream n=0</td>
<td>0.2015</td>
<td>0.1610</td>
<td>0.1375</td>
<td>0.1349</td>
</tr>
<tr>
<td>Downstream n=1</td>
<td>Cut off</td>
<td>0.0787</td>
<td>0.2140</td>
<td>0.1698</td>
</tr>
<tr>
<td>Upstream n=0</td>
<td>0.1363</td>
<td>0.0143</td>
<td>0.0313</td>
<td>0.0195</td>
</tr>
<tr>
<td>Upstream n=1</td>
<td>Cut off</td>
<td>0.0370</td>
<td>0.0763</td>
<td>0.0586</td>
</tr>
</tbody>
</table>

Unsteady Lift Coefficient

Acoustic Coefficients
Effect of Frequency
Mean Flow 4

\( M_o = 0.3536, \ M_r = 0.1, \ M_\Omega = 0.1, \ \frac{r_h}{r_t} = 0.6667, \ \text{and} \ \alpha_r = 0 \)

Unsteady Lift Coefficient

Acoustic Coefficients
\( m = -8, \ n = 0.1 \)
Effect of the Upstream Disturbance radial component

\[ M_o = 0.3536, \ M_I = 0.1, \ M_\Omega = 0.1, \ \frac{r_h}{r_t} = 0.6667, \ \text{and} \ \omega = 3\pi \]

<table>
<thead>
<tr>
<th>Mode ( m = -8 )</th>
<th>2-D Disturbance</th>
<th>3-D Disturbance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Downstream ( n = 0 )</td>
<td>0.1471</td>
<td>0.1094</td>
</tr>
<tr>
<td>Downstream ( n = 1 )</td>
<td>0.0908</td>
<td>0.0617</td>
</tr>
<tr>
<td>Upstream ( n = 0 )</td>
<td>0.0185</td>
<td>0.0143</td>
</tr>
<tr>
<td>Upstream ( n = 1 )</td>
<td>0.0498</td>
<td>0.0418</td>
</tr>
</tbody>
</table>

Unsteady Lift Coefficient

Acoustic Coefficients
Scattering of Acoustic versus Vortical Disturbance

\[ M_0 = 0.3536, \quad M_\Gamma = 0.1, \quad M_\Omega = 0.1, \quad \frac{r_h}{r_t} = 0.6667, \text{ and } \omega = 3\pi \]

<table>
<thead>
<tr>
<th>Mode m=-8</th>
<th>Vortical Disturbance</th>
<th>Acoustic Disturbance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Downstream n=0</td>
<td>0.1471</td>
<td>0.8608</td>
</tr>
<tr>
<td>Downstream n=1</td>
<td>0.0908</td>
<td>0.1458</td>
</tr>
<tr>
<td>Upstream n=0</td>
<td>0.0185</td>
<td>0.0718</td>
</tr>
<tr>
<td>Upstream n=1</td>
<td>0.0498</td>
<td>0.0775</td>
</tr>
</tbody>
</table>

Unsteady Lift Coefficient

Acoustic Coefficients
Comparison of Successive Iterations
Three-Dimensional Effects
Comparison With Strip Theory

![Graph showing sectional lift coefficient vs. span with lines labeled 3D and LINC]
Effect of Swirl

- Left graph: Sectional Lift Coefficient at $\alpha_{\text{nom}}$
  - $\alpha=0^\circ$
  - $\alpha=15^\circ$
  - $\alpha=30^\circ$
  - $\alpha=45^\circ$

- Right graph: Acoustic Pressure (1st mode)
  - $\alpha=0^\circ$
  - $\alpha=15^\circ$
  - $\alpha=30^\circ$
  - $\alpha=45^\circ$

- Bottom right graph: Acoustic Pressure (2nd mode)
  - $\alpha=0^\circ$
  - $\alpha=15^\circ$
  - $\alpha=30^\circ$
  - $\alpha=45^\circ$
Effect of Blade Twist

![Graphs showing the effect of blade twist on lift coefficient.]
Conclusions

• Swirl affects the impedance of the duct and the number of cut-on acoustic modes. The spinning modes are not symmetric.
• Strip theory gives good approximation as long as there is no acoustic propagation.
• Resonant conditions in strip theory are much more pronounced than for 3D, i.e., lift variation in 3D is much smoother along span.
• Discrepancies between strip theory and 3D calculations increase with the reduced frequency: it is a high frequency phenomenon.