Aeroacoustic and Aerodynamics of Swirling Flows*

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OVERVIEW OF PRESENTATION

- Disturbances in Swirling Flows
- Normal Mode Analysis
- Application to Computational Aeroacoustics
- Vortical Disturbances
- Aerodynamic and Acoustic Blade Response
- Conclusions
Swirling Flow in a Fan
Issues For Consideration

- Effect of swirl on aeroacoustics and aerodynamics?
- Can we consider separately acoustic, vortical and entropic disturbances?
- How does swirl affect sound propagation (trapped modes)?
- How do vortical disturbances propagate?
- How strong is the coupling between pressure, vortical and entropic modes?
- What are the conditions for flow instability?
- What are the boundary conditions to be specified?
Scaling Analysis

- **Acoustic phenomena:**
  - Acoustic frequency: \( nB \Omega \)
  - Rossby number = \( \frac{nB \Omega r_t}{c_0} \gg 1 \)

- **Convected Disturbances:**
  - Convection Frequency ~ Shaft Frequency \( \Omega \)
  - Rossby number = \( \frac{\Omega r_t}{U_x} \approx O(1) \)
  - Wakes are distorted as they convect at different velocity. Centrifugal and Coriolis accelerations create force imbalance which modifies amplitude and phase and may cause hydrodynamic instability.
Mathematical Formulation

- Linearized Euler equations
- Axisymmetric swirling mean flow

\[ \tilde{U}(\tilde{x}) = U_x(x, r)\tilde{c}_x + U_s(x, r)\tilde{c}_\theta \]

- Mean flow is obtained from data or computation
- For analysis the swirl velocity is taken

\[ U_s = \Omega r + \frac{\Gamma}{r} \]

- The stagnation enthalpy, entropy, velocity and vorticity are related by Crocco’s equation

\[ \nabla H = TVS + U \times \zeta \]
Normal Mode Analysis
A normal mode analysis of linearized Euler equations is carried out assuming solutions of the form

\[ f(r)e^{i(-\omega t + m_n \theta + k_{mn} x)} \]

- Eigenvalue problem is not a Sturm-Liouville type
- A combination of spectral and shooting methods is used in solving this problem
  - Spectral method produces spurious acoustic modes
  - Shooting method is used to eliminate the spurious modes and to increase the accuracy of the acoustic modes
Comparison Between the Spectral and Shooting Methods

$M_x=0.55, M_\Gamma=0.24, M_\Omega=0.21, \omega=16$, and $m=-1$
Effect of Swirl on Eigenmode Distribution

$M_{xm} = 0.56, M_\Gamma = 0.25, M_\Omega = 0.21$
Pressure Content of Acoustic and Vortical Modes

\[ M_x=0.5, \ M_\Gamma=0.2, \ M_\Omega=0.2, \ \omega=2\pi, \ \text{and} \ m=-1 \]
Summary of Normal Mode Analysis

Normal Modes

- Pressure-Dominated Acoustic Modes
  - Propagating
  - Decaying
    - Nonreflecting Boundary Conditions
- Vorticity-Dominated Nearly-Convected Modes
  - Singular Behavior

Nonreflecting Boundary Conditions
Accurate nonreflecting boundary conditions are necessary for computational aeroacoustics.
\\[ p(\vec{x},t) = \int \sum_{\nu=-\infty}^{\infty} \sum_{n=0}^{\infty} c_{mn} p_{mn}(\omega, r) e^{i(-\omega t + \nu \theta + k_{mn} x)} d\omega \]
Nonreflecting Boundary Conditions (Cont.)

\[ p(\vec{x}, t) = \sum_{\nu = -M/2}^{M/2} \sum_{n=0}^{N} c_{mn} p_{mn}(r) e^{i(\omega t + m_\nu \theta + k_{mn} x)} \]

\[ \begin{bmatrix} p \end{bmatrix}_{L-1} = [\mathcal{R}_{L-1}] [c] \]

\[ \begin{bmatrix} p \end{bmatrix}_L = [\mathcal{R}_L] [c] \]

\[ \begin{bmatrix} p \end{bmatrix}_L = [\mathcal{R}_L] [\mathcal{R}_{L-1}]^{-1} [\begin{bmatrix} p \end{bmatrix}_{L-1}] \]
Application to Computational Aeroacoustics
Test Problems for Acoustic Waves

- Acoustic waves and/or a combination of acoustic and vortical waves are imposed upstream of an annular duct with swirling mean flow and nonreflecting boundary condition applied downstream.

Quieting the skies: engine noise reduction for subsonic aircraft
Advanced subsonic technology program. NASA Lewis research center, Cleveland, Ohio

Acoustic and/or Vortical Mode

Nonreflecting Boundary conditions
Acoustic Normal Mode Spectrum

$M_x = 0.5$, $M_\Gamma = 0.2$, $M_\Omega = 0.2$, $\omega = 2\pi$, and $m = -1$
Density and Velocity Distribution in Uniform Flow

\[ k_{-1,1} = 4.1077 \]
Density and Velocity Distribution in Swirling Flow

First Propagating Acoustic Mode

\[ k_{-1,1} = 4.3942 \]
Density and Velocity Distribution in Swirling Flow

Second Propagating Acoustic Mode

\[ k_{-1,2} = -2.4639 \]
Density and Velocity Distribution in Swirling Flow

Acoustic & Vortical Modes

$$k_{-1,1} = 4.3942$$
$$k_{-1,3} = 11.7626$$
Sensitivity of Numerical Solutions to Accuracy of Eigenvalue

\[ M_x=0.5, \quad M_\Gamma=0.2, \quad M_\Omega=0.2, \quad \omega=2\pi, \quad \text{and} \quad m=-1 \]
Vortical Disturbances
Initial Value Solution

\[ u(x, r, \theta, t) = \int_{\infty}^{\infty} \sum_{m=\infty}^{\infty} A_m(x, r) e^{i(\alpha x + m \theta - \omega t)} d\omega \]

\[ \frac{D}{Dt} (\alpha x + m \theta - \omega t) = 0 \]
Wake Distortion by Swirl
Accelerating axial flow

\[ U(x, r) = 85 \left( 1 + \frac{\gamma}{L} \right) e_x + \frac{50}{r} e_\theta, \quad m = 10, \quad \omega = 5000 \]
Effect of viscosity

- Small scales are most affected by viscosity.
  - For large modal number $m$ (equivalent to wave-number), viscous effects are large.
- Rapid-distortion theory assumes viscosity as a source term modifying the evolution process.
  - Slip/Non-slip boundary conditions were tested.
Effect of Reynolds number on the modes

\[ U(x, r) = 85 e_x + \frac{50}{r} e_\theta, \ m = 10 \quad \text{and} \quad m = 20, \ \omega = 5000 \]

\[ \beta = \frac{1}{\text{Re}_t \rho_o U_o} \]

\[ \chi \approx O \left( \exp \left( -\frac{\beta m^2}{r^2} x \right) \right) \]

\[ \text{Re} = 10,000 \]
Aerodynamic and Acoustic Blade Response
Aerodynamic and Acoustic Blade Response

- Swirling Mean flow + disturbance
  - Rapid distortion theory "disturbance propagation"
    - Normal mode analysis "construction of nonreflecting boundary conditions"
      - Source term on blades
        - Blade unsteady loading & radiated sound field
          - Non-reflecting boundary conditions

- Linearized Euler model
Two schemes are developed:

- **Primitive variable approach**
  - Pseudo Time Formulation.
  - Lax-Wendroff Scheme.
- **Splitting velocity field approach**
  - Help understand physics.
  - Computational time requirements reduced.
  - No singularity at leading edge.
  - Implicit scheme leads to large number of equations which must be solved using an iterative method.
  - Parallelization significantly reduces computational time.
Benchmark Test Problem

\[ v_\theta(r, \theta, x) = \alpha U_x e^{i(\omega x + B\theta + h(r))} \]

\[ h(r) = -\frac{2\pi q}{B} \left( \frac{r - r_{\text{hub}}}{r_{\text{tip}} - r_{\text{hub}}} \right) \]
## Parameters for Benchmark Test Problem

<table>
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<th>窄径域</th>
<th>全径域</th>
<th>数据</th>
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<tr>
<td>(\frac{r_{\text{tip}}}{r_{\text{hub}}})</td>
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<td>(\omega)</td>
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<td>(V) (转子叶片)</td>
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<td>(C) (弦长)</td>
<td>(2\pi/V)</td>
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<td>(L) (长度)</td>
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Primitive Variable Approach

- Linearized Euler Equations
- Pseudo Time Formulation

\[
\begin{bmatrix}
0 \\
0 \\
\rho' u_x \\
\rho' u_\theta \\
\rho' u_r \\
\rho' \theta
\end{bmatrix} = 0
\]

Lax-Wendroff Scheme
Unsteady Pressure Jump Across the Blade for q=1 at Different Spanwise Locations

Primitive Variable Approach

ND: real part -, imaginary part --; Schulten: real part -., imaginary part …
Unsteady Pressure Jump Across the Blade for $q=3$ at Different Chordwise Locations

Primitive Variable Approach

ND: real part -, imaginary part --; Schulten: real part -.-, imaginary part …
Acoustic Coefficients for Mode (1,0) at Different Gust Spanwise Wavenumbers

Primitive Variable Approach

Upstream

Downstream
Acoustic Coefficients for Mode (1,1) at Different Gust Spanwise Wavenumbers

Primitive Variable Approach

Upstream

Downstream
## Magnitude of the Downstream Acoustic Coefficients

### Primitive Variable Approach

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<th>k</th>
<th>μ</th>
<th>Namba</th>
<th>Schulten</th>
<th>ND</th>
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# Magnitude of the Upstream Acoustic Coefficients

## Primitive Variable Approach

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<th>Namba</th>
<th>Schulten</th>
<th>ND</th>
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</tbody>
</table>


**Splitting Velocity Approach**

\[ \mathbf{u} = \mathbf{u}^R + \nabla \phi \]

\[ p' = -\rho_o \frac{D_o \phi}{Dt} \quad \text{where} \quad \frac{D_o}{Dt} = \frac{\partial}{\partial t} + \mathbf{U}_o \cdot \nabla \]

\[
\begin{align*}
\frac{D_o}{Dt} & \left( \frac{1}{c_o^2} \right) \frac{D_o \phi}{Dt} - \frac{1}{\rho_o} \nabla \cdot \left( \rho_o \nabla \phi \right) = \frac{1}{\rho_o} \nabla \cdot \left( \rho_o \mathbf{u}^R \right) - \frac{\partial s'/\partial t}{2c_p} \\
\frac{D_o}{Dt} \mathbf{u}^R + \left( \mathbf{u}^R \cdot \nabla \right) \mathbf{U} &= -\left( \nabla \times \mathbf{U} \right) \times \nabla \phi - \frac{D_o \phi}{Dt} \frac{\nabla s_o}{c_p} \\
\frac{D_o}{Dt} S' + \left( \mathbf{u}' \cdot \nabla \right) s_o &= 0
\end{align*}
\]
Narrow Annulus

- $\omega_{rm}=7.55$
- Grid sensitivity study
- Pressure difference compared to LINC.

<table>
<thead>
<tr>
<th></th>
<th>Upstream</th>
<th>Downstream</th>
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</thead>
<tbody>
<tr>
<td>m=-8</td>
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<tr>
<td>Namba</td>
<td>$7.58 \times 10^{-3} - 1.81 \times 10^{-3}i$</td>
<td>$-1.12 \times 10^{-2} + 5.68 \times 10^{-3}i$</td>
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<tr>
<td>Schulten</td>
<td>$7.36 \times 10^{-3} - 2.453 \times 10^{-3}i$</td>
<td>$-9.95 \times 10^{-3} + 5.87 \times 10^{-3}i$</td>
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<tr>
<td>ND</td>
<td>$7.03 \times 10^{-3} - 3.86 \times 10^{-3}i$</td>
<td>$-9.67 \times 10^{-3} + 6.58 \times 10^{-3}i$</td>
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Full Annulus Case: Pressure jump for $q=0$, $\omega_{rm}=9.396$. Comparison with Schulten

Splitting Approach
Spanwise Pressure Jump for $q=3$, $\omega_{rm} = 9.396$. Comparison with Schulten

Splitting Approach

\[ \Delta P \text{ at } x/c=0.05 \]

\[ \Delta P \text{ at } x/c=0.2 \]

\[ x/c=0.9 \]
Upstream & Downstream Acoustic Coefficients.

Splitting Approach

Comparison of upstream acoustic modes

Comparison of downstream acoustic modes
Lift Coefficient for $q=0, 3$ versus $\omega$ and radius

Splitting Approach
Full Annulus Lift Distribution
Comparison with Strip Theory

Splitting Approach
Meridian Plane Approximation for Mean Flow (2D Cascade)

Actual Meanflow
20° stagger, M=0.3

Meridianal Meanflow
Unsteady Lift Comparison
Actual and Meridional Meanflows

Low Loading $C_l=0.20$

High Loading $C_l=0.92$
Conclusions

- For swirling flows, two families of normal modes exist: pressure-dominated nearly-sonic, and vorticity-dominated nearly-convected modes.
- Nonreflecting boundary conditions were derived, implemented, and tested for a combination of acoustic and vorticity waves.
- An initial-Value formulation is used to calculate incident gusts.
- Two schemes (primitive variable and splitting) have been developed for the high frequency aerodynamic and acoustic blade response. Results are in good agreement with boundary element codes.
- A meridian approximation of the mean flow gives “surprising” good unsteady results for 2D cascades.
Future Work

- The numerical code will be used to study unloaded annular cascades in swirling flows.
- Method is under development for loaded annular cascades in swirling flows using a meridian approach.
- Parallelization will significantly reduce computational time making it possible to treat broadband noise.
- Express results in term of the acoustic power radiated.