2004 ASME Rayleigh Lecture

Fluid-Structure Interaction and Acoustics

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“Rarely does one find a mass of analysis without illustrations from experience.”

Rayleigh
Overview of Lecture

- Sound Generated by Fluid-Structure Interaction Phenomena
- Linear Acoustic, Entropic and Vortical Mode Splitting
- Non-uniform Flow Effects
- Application:
  - Aircraft Turbofan Interaction Noise
  - Marine Propellers with Elastic Ducts
- Conclusions
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When Acoustics is Important?

- High Speed
  - Aeronautics
  - Transportation
  - Energy Production
- Stealth
  - Submarines
- Structural Damage
- High Perceived Noise Level
  - Frequency Range
  - Intensity
Generic Problem
Airfoil in Nonuniform Flow

Unsteady Nonuniform Upstream Flow

Unsteady Pressure in the Near Field

Vortex Shedding in the Wake

Acoustic Radiation in the Far Field
Aeroacoustics and Unsteady Aerodynamics

- Lighthill's acoustic analogy.

- Sound is the far-field signature of the unsteady flow.

- The aeroacoustic problem is similar to that of forced vibration but with emphasis on the far-field. It is a much more difficult computational problem whose outcome depends on preserving the far-field wave form with minimum dispersion and dissipation.

- Inflow/outflow nonreflecting boundary conditions must be derived to complete the mathematical formulation as a substitute for physical causality.
Kovasnyay Modes
(1953)

- Acoustics
- Entropy
- Vorticity
Splitting Theorem for Uniform flow

\[ V(\mathbf{x}, t) = U + u(\mathbf{x}, t) \]

\[ u = u^{(R)} + \nabla \Phi \]

\[ \frac{D_0 u^{(R)}}{Dt} = 0 \]

\[ \frac{D_0 s'}{Dt} = 0 \]

\[ \left( \frac{D_0^2}{Dt^2} - c_0^2 \nabla^2 \right) \Phi = 0 \]

\[ \frac{D_0}{Dt} \equiv \frac{\partial}{\partial t} + U \cdot \nabla \]
Disturbances in Uniform Flows

Splitting Theorem:

The flow disturbances can be split into distinct potential (acoustic), vortical and entropic modes obeying three independent equations:

- The vortical velocity is solenoidal, purely convected and completely decoupled from the pressure fluctuation.
- The potential (acoustic) velocity is directly related to the pressure fluctuations.
- The entropy is purely convected and only affects the density through the equation of state.
- Coupling between the vortical and potential velocity occurs only along the body surface.
- Upstream conditions can be specified independently for various disturbances.
Equations for Linear Aerodynamics

- **Vortical Mode:** 
  \[ \vec{u}_v = \vec{u}_\infty (\vec{x} - \vec{U} \imath) \]

- **Harmonic Component**
  \[ \vec{u}_g = \vec{a} e^{i(k \cdot \vec{x} - \omega t)} \]

- **Potential Mode:**
  \[ \left( -\frac{1}{c_0^2} \frac{D_0}{Dt} \frac{2}{2} - \nabla^2 \right) \phi = 0 \]

- **Boundary Conditions:** impermeability along blade surface, Kutta condition at trailing edge, allow for wake shedding in response to gust.
Issues Associated with Nonuniform Flows

- Linear vs Nonlinear Analysis:
  - Uniform mean flow, RDT, fully nonlinear.
- Inflow Disturbance Description:
  - Can we consider separately acoustic, vortical and entropic disturbances?
- What are the upstream boundary conditions to be specified?
- What are the conditions for flow instability?
- What is the effect of high frequency?
  - Can we still apply The Kutta condition?
  - Difficult computational issues but help from asymptotics?
- What are transonic flow effects?
- Conservation relations for acoustic intensity and power?
- Turbulence modification.
- Coupling with structural modes.
Linear Versus Nonlinear
Mode Interaction and Coupling

Acoustics

Heat Spots
Density change
Heat convection
Vorticity generation

Entropy

Heat Spots
Instability
Vorticity generation

Vorticity

Distortion
Wave steepening
Mean flow gradients
Vortex stretching
Instability

Refraction & Reflection
Diffraction
Dispersive waves
No energy conservation
Vortex Traveling around an airfoil

Linear Theory

Comparison Between Theory and Computation:
Surface Pressure Induced By the Motion of a Vortex Near the Trailing Edge
As the vortex travels near the trailing edge it is no longer convected by the mean flow. Its trajectory crosses the undisturbed mean flow. This increases the amount of fluid energy converted into acoustic energy. The acoustic power scales with $M^3$, much higher than that predicted by a dipole ($M^6$).
Cascade Flow with local Regions of Strong Interaction
This leads to a singular behavior for the gust at the leading edge.
Comparison between Linear and Nonlinear Analyses for Subsonic Flows

Magnitude (a) and Phase (b) of the first Harmonic Unsteady Pressure Difference Distribution for the subsonic Cascade Undergoing an In-Phase Torsional Oscillation of Amplitude $a=2\alpha$ at $k_1=0.5$ about Midchord; $M=0.7$; ..., Linear Analysis; ...., Nonlinear Analysis.
Comparison between Linear and Nonlinear Analyses for Transonic Flows

Magnitude (a) and Phase (b) of the first Harmonic Unsteady Pressure Difference Distribution for the Supersonic Cascade Undergoing an In-Phase Torsional Oscillation of Amplitude $a=2^\circ$ at $k_1=0.5$ about Midchord; $M=0.8$. 

Acoustic Blockage
Summary

- RDT unsteady analysis yields good results for subsonic flows.
- RDT unsteady analysis is adequate for transonic flows with local Mach number not exceeding 1.3.
- Strong nonlinear effects resulting from shock boundary layer interaction are significant as the local Mach number exceeds 1.3-1.4.
Aircraft Noise

- Fan noise sources:
  - (high frequency phenomena)
  - Rotor/stator interaction
  - Boundary layers and ingested turbulence
  - Rotor noise

![Diagram showing noise sources](image)
Typical Fan Sound Power Spectra

Subsonic Tip Speed

Supersonic Tip Speed
Anatomy of a Turbo-fan Engine

1 = far-field inlet  
2 = inlet plane  
3 = inlet duct  
4 = fan rotor  
5 = rotor-stator gap  
6 = outlet guide vanes  
7 = engine section stator  
8 = bypass duct  
9 = cold jet nozzle  
10 = far-field cold exit  
11 = low-pressure compressor  
12 = high-pressure compressor  
13 = combustion chamber  
14 = high-pressure turbine  
15 = low-pressure turbine  
16 = turbine exhaust duct  
17 = hot jet nozzle  
18 = far-field hot exit
Swirling Flow Phenomena
Schematic of Rotor Wake Phenomena
Wake Distortion by Swirl
Scaling Analysis

- Two Length Scales:
  - Body Length Scale: $\ell$
  - Turbulence Integral Scale: $\Lambda \ll \ell$

- Two Velocities:
  - Convection velocity: $U$
  - Speed of sound: $c_0$

- Aerodynamic Frequencies:
  - $\omega^* = \frac{\omega \ell}{U} \gg 1$
  - $\omega' = \frac{\omega \Lambda}{U} = O(1)$

- Acoustic Frequencies
  - $\tilde{\omega} = \frac{\omega \ell}{c_0}$

- Fast and Slow Variables:
  - $\tilde{x}^* = \frac{\tilde{x}}{\ell}$
  - $\tilde{x} = \frac{\tilde{x}}{\Lambda}$
Can high frequency help?

Can we turn the scourge to advantage?
Linearized Euler Equations

\[ \mathbf{V}(\mathbf{x},t) = \mathbf{U}(x) + \mathbf{u}(x,t) \]

Continuity:

\[ \frac{D}{D_0 t} \left( \frac{\rho'}{\rho_0} \right) + \frac{1}{\rho_0} \nabla \cdot (\rho_0 \mathbf{u}) = 0 \]

Momentum:

\[ \frac{D\mathbf{u}}{D_0 t} + \mathbf{u} \cdot \nabla \mathbf{U} = -\nabla \left( \frac{p'}{\rho_0} \right) + \frac{p'}{\rho_0} \frac{\nabla s_0}{c_p} - \frac{s'}{c_p} \frac{\nabla p_0}{\rho_0} \]

Energy:

\[ \frac{D s'}{D_0 t} + \mathbf{u} \cdot \nabla s_0 = 0 \]

Thermodynamics:

\[ s' = c_v \frac{p'}{p_0} - c_p \frac{\rho'}{\rho_0} \]
Governing Equations in Splitting Form

\[ \mathbf{u}(x,t) = \mathbf{u}^{(R)} + \nabla \phi + \frac{s'}{2c_p} \mathbf{U} \]

\[ \frac{D_o \mathbf{u}^{(R)}}{Dt} + \mathbf{u}^{(R)} \cdot \nabla \mathbf{U} - \frac{\mathbf{U}}{2c_p} \cdot (\mathbf{u}^{(R)} \cdot \nabla S_0) = \nabla \times (\nabla \times \mathbf{U}) - \frac{D_o \phi}{Dt} \frac{\nabla S_0}{c_p} + \frac{\mathbf{U}}{2c_p} (\nabla \phi \cdot \nabla S_0) \]

\[ \frac{D_o}{Dt} \left( \frac{1}{c_0^2} \frac{D_o \phi}{Dt} \right) - \frac{1}{\rho_0} \nabla (\rho_0 \nabla \phi) - \frac{\nabla \phi \cdot \nabla S_0}{c_p} = \frac{1}{\rho_0} \nabla (\rho_0 \mathbf{u}^{(R)}) + \frac{\mathbf{u}^{(R)} \cdot \nabla S_0}{c_p} + \frac{\mathbf{U} \cdot \nabla S'}{2c_p} \]

\[ \frac{D_o s'}{Dt} + \mathbf{u} \cdot \nabla S_0 = 0 \]

\[ s' - c_v \frac{p'}{p_0} + c_p \frac{\rho'}{\rho_0} = 0 \]

\[ p' = -\rho_0 \frac{D_o}{Dt} \phi \]
Normal Mode Analysis
Normal Mode Analysis

- A normal mode analysis of linearized Euler equations is carried out assuming solutions of the form
  \[ f(r)e^{i(-\omega t + m_\nu \theta + k_{mn} x)} \]

- Eigenvalue problem is not a Sturm-Liouville type and singular for vanishing
  \[ \Lambda_{mn} = \frac{D_0}{Dt} = -\omega + k_{mn} U_x + \frac{mU_s}{r} \]

- A combination of spectral and shooting methods is used in solving this problem
Mode Spectrum
Spectral and Shooting Methods

P&W mean flow data, $\omega=16$, and $m=-1$
Coupling Between Pressure, Vorticity, and Entropy

Non-isentropic flow $M_x=0.3$, $M_\Gamma=0.3$, $M_\Omega=0.3$, $\omega=10$, and $m=2$
Effect of Swirl on Eigenmode Distribution

\[ M_{xm}=0.56, \ M_\Gamma=0.25, \ M_\Omega=0.21 \]
Upstream Representation of Disturbances
Upstream Disturbance Representation

\[ \vec{u} = \vec{u}_R + \nabla \Phi \]

Vortical velocity is solenoidal \[ \nabla \cdot \vec{u}_R = 0 \]
Upstream Disturbance Representation (2)

Normal Mode Analysis → Vortical Modes $p_v \cong 0$ → Continuity and Momentum

Acoustic Modes → Compatibility Condition

$u_i = u_a + u_v$

$P_i = P_a$

Incident Disturbance

Computational Domain

Scattered Acoustics

Non-Reflecting B.C.
Nonreflecting Boundary Conditions
Nonreflecting Boundary Conditions

- Pressure at the boundaries is expanded in terms of the acoustic eigenmodes.

\[
p(\vec{x}, t) = \int_{\omega} \sum_{v=-\infty}^{\infty} \sum_{n=0}^{\infty} c_{mn} \ p_{mn}(\omega, r) e^{i(-\omega t + m, \theta + k_{mn} x)} d\omega
\]

- Only outgoing modes are used in the expansion.
- Group velocity is used to determine outgoing modes.
Nonreflecting Boundary Conditions (Cont.)

\[ p(\vec{x}, t) = \sum_{\nu = -M/2}^{M/2} \sum_{n=0}^{N} c_{mn} p_{mn}(r) e^{i(-\omega t + m \nu \theta + k_{mn} x)} \]

\[ [p] = [\mathcal{R}] [c] \]

Computational Domain

\[ [p]_{L-1} = [\mathcal{R}]_{L-1} [c] \]

\[ [p]_L = [\mathcal{R}]_L [c] \]

\[ [p]_L = [\mathcal{R}]_L [\mathcal{R}]_{L-1}^{-1} [p]_{L-1} \]
Aerodynamic-Aeroacoustic Model

Disturbance propagation

Swirling mean flow + disturbance

Rapid distortion + Multiple scale

Normal mode analysis

Broadband Spectra

Source term on blades

Blade unsteady loading & radiated sound field

Nonreflecting inflow/outflow conditions

Euler model
Schematics of the Computational Domain

High frequency asymptotics come to help

Domain Decomposition of the Computational Domain

The Annular Cascade
Incidence Disturbance Coupling with duct Modes
B=16, V=24, c=2\pi/24, r_h/r_t=0.5, L=3c

- Often the largest amplitudes of the incident vortical disturbances occur in either the hub and tip regions of the duct where viscous effects result in significant intensification of the wake.

- Hub-dominated incident disturbance:

\[ a_{m_g}^{(u)} = \cos \frac{\pi}{2} \frac{r - r_h}{r_t - r_h} \]

- Tip-dominated incident disturbance:

\[ a_{m_g}^{(u)} = \cos \frac{\pi}{2} \frac{r_t - r}{r_t - r_h} \]
Profile of the First Downstream and Upstream Modes

- Uniform Flow, -- Rigid body swirl, -. - Free vortex swirl

\[ \omega = 2.5 \pi \]
Magnitude of the upstream and downstream acoustic modes $\omega=2.5\pi$

| n  | $|c^+_{mn}|$ | $|c^-_{mn}|$ | $|c^+_{mn}|$ | $|c^-_{mn}|$
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>CASE 1</td>
<td>CASE 2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>hub-dominated</td>
<td>tip-dominated</td>
<td>hub-dominated</td>
<td>tip-dominated</td>
<td></td>
</tr>
<tr>
<td>Uniform Flow $M_x=0.4$ $M_\Omega=0.0$ $M_r=0.0$</td>
<td>1</td>
<td>0.048 (ht)</td>
<td>0.040 (ht)</td>
<td>0.059 (tt)</td>
</tr>
<tr>
<td>Rigid Body Swirl $M_x=0.4$ $M_\Omega=0.0$ $M_r=0.0$</td>
<td>2</td>
<td>0.099 (hh)</td>
<td>0.073 (hh)</td>
<td>0.009 (th)</td>
</tr>
<tr>
<td>Potential Swirl $M_x=0.4$ $M_\Omega=0.0$ $M_r=0.0$</td>
<td>1</td>
<td>0.083 (ht)</td>
<td>0.018 (hh)</td>
<td>0.102 (tt)</td>
</tr>
<tr>
<td>2</td>
<td>0.110 (hh)</td>
<td>0.025 (hh)</td>
<td>0.020 (th)</td>
<td>0.049 (th)</td>
</tr>
</tbody>
</table>

$M_x$, $M_\Omega$, and $M_r$ are the mass flow rates through the hub, tip, and ring, respectively.
Comparison of Acoustic Response of 2D and 3D Cascades to a Harmonic Excitation

- $\omega=30.50$
- $m_g=16$
- $n_g=0$
- $h_r/h_t = 0.6$
- $M=0.5$
- Stagger= 0$^\circ$ and 30$^\circ$
- Swirl $M_\Gamma = M_\Omega = 0.125$ (at $r_m$)
- Blade Number=24
- Spacing=1
Acoustic Response and Axial Wave Number of a 2D Linear Cascade
$\omega=30.50$, $m_g=16$, $n_g=0$, $M=0.5$, stagger=30°, $B=24$, $s/c =1$

### Downstream

| $m$ | $n$ | $k_{mn}$ | $|c_{mn}|$ |
|-----|-----|----------|-----------|
| 16  | 0   | 7.6081   | 0.0542    |
| -8  | 0   | 17.6352  | 0.0375    |
| -32 | 0   | 2.1685   | 0.0708    |

### Upstream

| $m$ | $n$ | $K_{mn}$ | $|c_{mn}|$ |
|-----|-----|----------|-----------|
| 16  | 0   | 29.9700  | 0.0189    |
| -8  | 0   | 46.4075  | 0.0371    |
| -32 | 0   | 37.2995  | 0.0177    |

3 propagating modes ($n=n_g$)
Top View of the Acoustic Pressure of a 2D Linear Cascade
Acoustic Response and Axial Wave Number of a 3D Annular Cascade  
\( \omega = 30.50, \, m_g = 16, \, n_g = 0, \, M = 0.5, \, \text{stagger} = 0^\circ, \, B = 24, \, s/c = 1, \, h_r/h_t = 0.6 \)

| m   | n   | \( k_{mn} \) | \( |c_{mn}| \) |
|-----|-----|-------------|-------------|
| 16  | 0   | 12.1905     | 0.1090      |
| 16  | 1   | 8.9474      | 0.0595      |
| 16  | 2   | 5.7227      | 0.0374      |
| 16  | 3   | -0.5668     | 0.0196      |
| -8  | 0   | 15.4739     | 0.0449      |
| -8  | 1   | 14.2355     | 0.0216      |
| -8  | 2   | 11.7102     | 0.0189      |
| -8  | 3   | 6.6571      | 0.0079      |
| -8  | 4   | -3.3952     | 0.0041      |
| -32 | 0   | -7.1684     | 0.1449      |

| m   | n   | \( k_{mn} \) | \( |c_{mn}| \) |
|-----|-----|-------------|-------------|
| 16  | 0   | -32.7664    | 0.0293      |
| 16  | 1   | -39.0561    | 0.0202      |
| 16  | 2   | -42.2807    | 0.0696      |
| 16  | 3   | -45.5238    | 0.0414      |
| -8  | 0   | -29.9381    | 0.0032      |
| -8  | 1   | -39.9905    | 0.0074      |
| -8  | 2   | -45.0435    | 0.0134      |
| -8  | 3   | -47.5688    | 0.0114      |
| -8  | 4   | -48.8073    | 0.0125      |
| -32 | 0   | -26.1648    | 0.1560      |

10 propagating modes
Top View of the Acoustic Pressure of a 3D Annular Cascade
Zero-Stagger
Acoustic Response and Axial Wave Number of a 3D Annular Cascade
\( \omega=30.50, \ m_g=16, \ n_g=0, \ M=0.5, \ \text{stagger}=30^\circ, \ B=24, \ s/c =1, \ h_r/h_t = 0.6 \)

| m  | n  | \( k_{mn} \) | \( |c_{mn}| \) |
|----|----|-------------|-------------|
| 16 | 0  | 9.8459      | 0.0519      |
| 16 | 1  | 4.4321      | 0.0736      |
| 16 | 2  | -2.1739     | 0.1162      |
| -8 | 0  | 17.8059     | 0.0659      |
| -8 | 1  | 16.6875     | 0.0356      |
| -8 | 2  | 14.3758     | 0.0115      |
| -8 | 3  | 9.8157      | 0.0098      |
| -8 | 4  | 1.3140      | 0.0028      |
| -32| 0  | 7.5623      | 0.0882      |
| -32| 1  | -0.2123     | 0.0382      |
| -32| 2  | -10.8543    | 0.0436      |

| M  | n  | \( k_{mn} \) | \( |c_{mn}| \) |
|----|----|-------------|-------------|
| 16 | 0  | -20.4406    | 0.1097      |
| 16 | 1  | -26.6919    | 0.0794      |
| 16 | 2  | -30.8843    | 0.0811      |
| -8 | 0  | -30.1154    | 0.0043      |
| -8 | 1  | -38.5356    | 0.0125      |
| -8 | 2  | -42.7368    | 0.0093      |
| -8 | 3  | -44.9057    | 0.0281      |
| -8 | 4  | -48.4882    | 0.0235      |
| -32| 0  | -25.7269    | 0.0267      |
| -32| 1  | -33.9841    | 0.0241      |
| -32| 2  | -38.6116    | 0.0148      |

11 propagating modes
Top View of the Acoustic Pressure of a 3D Annular Cascade
30°- Stagger
Summary for Tonal Noise Response of an Annular Cascade

- The nonuniform swirling flow changes the physics of scattering in 3 major ways:
  - It increases the number of acoustic modes in the duct.
  - It changes their duct radial profile.
  - It causes significant amplitude and radial phase variation of the incident disturbances.
- The higher number of cut-on modes is due to the fact that the acoustic radial mode number, $n$, is no longer restricted to be equal to that of the upstream excitation, i.e., $n=n_g$.
- When the radial phase of the incident disturbance is different from that of the duct modes, weak scattering occurs.
- Analysis of the radial variation of the incident disturbance and duct modes can provide an indication of the efficiency of the scattering process.
- Results suggest that 2D models underestimate the acoustic power by 3db at moderately high frequencies.
Hanson’s Cascade
Scattered Acoustic Power versus Frequency

- $h_r/h_t = 0.6$
- $M = 0.5$
- Stagger = $0^\circ$ and $30^\circ$ (at $r_m$)
- Swirl : $M_\Gamma = M_\Omega = 0.125$ (at $r_m$)
- Blade Number = 45
- Spacing = 0.8
- Turbulence Integral Scale = 0.032 $r_{(0.85)}$
- Turbulence Level/Mean Velocity = 0.018
Acoustic Power Level versus Frequency for an Annular Cascade at $35^\circ$ Stagger
Comparison with 2D Strip Theory

$\omega=35, 1000$ Hz
High Frequency Case

High frequency asymptotics come again to help

- At high frequency, the inflow disturbance interaction with the cascade is dominated by local effects. Airfoil theory suggests that the acoustic pressure has a simple dependence on the frequency of the form $1/\omega^\alpha$.

- For $\omega > 1/\Lambda$, the turbulence spectral density also has a simple dependence on frequency and dominates the contribution to the scattered broadband energy.

\[
\begin{align*}
    c_{mn} & \sim \omega^{-1/2} \\
    \text{For } k & \gg \frac{1}{\Lambda} \\
    \text{Liepmann' spectrum :} & \\
    \Phi_{ij} & \sim \frac{1}{\omega^{4}}, \quad P(\omega) \sim \frac{1}{\omega^{3}} \\
    \text{DATA :} & \\
    \Phi_{ij} & \sim \frac{1}{\omega^{11/3}}, \quad P(\omega) \sim \frac{1}{\omega^{2.66}}
\end{align*}
\]
Reduction of Hanson’s Results (stagger 30°)

Line \( \sim 1/\omega^{2.54} \)

Universal law at very high frequency

1000Hz, \( \omega = 35 \).
Conclusions for Broadband Noise

- At moderate frequencies the 3D model yields higher level of noise.

- At high frequency ($\omega >> 1/ \Lambda$) asymptotic analysis and preliminary results suggest the scattered acoustic power, $P \sim 1/ \omega^\alpha$ with $2.3 < \alpha < 2.75$.

- This obviates the need to calculate the scattered acoustic power for the computationally intensive high frequencies.

- Comparison with experimental results is planned.
Tonal and Broadband Sources of Noise in a Ducted Propeller

- **Blade force: dipoles**
- **Structural vibration**
- **Centrifugal force due to swirl**
- **Axial and swirling motion**
- **Turbulent boundary layer**
- **Strong rotot blades-duct interaction**
- **Ingested turbulence**
- **Ribs**
- **Water**

**Water**

**W**

**U**
Issues for Consideration

- How does coupling the flow-propeller interaction to the elastic duct system affect the propeller distributed dipole sources?

- Determine Transfer Functions for Acoustic Radiation for Various Duct Conditions.

- What conditions may lead to strong coupling between hydrodynamic disturbances and the elastic duct?
Coupling the propeller with the system

- Duct boundary condition determines duct modes and blade hydrodynamic response (dipole strength and orientation)
  - Rigid duct: \( u_r = 0 \).
  - Elastic duct: duct impedance determines duct modes
    - Dynamical Stiffness: \( p' = D(a, h, E, \nu, \alpha, \omega) \zeta_r \)
    - Impedance: \( Z = R + iX = i \frac{D}{\omega} \)
    - Fluid mode relation: \( p' = \Pi(\alpha, \omega, a, \rho) u_r \)
    - Dispersion equation:

\[
\Pi(\alpha, \omega, a, \rho) = Z(a, h, E, \nu, \alpha, \omega)
\]
Natural in Vacuo Wave Numbers versus Frequency for Different Circumferential Mode Numbers $m=0,1,2,3$. 

- **Flexure**: $m=0$
- **Torsion**
- **Compression**: $m=2$
- **m=1**
- **m=3**
Copper/Water Dispersion Relation
Submerged Duct
Copper/Water Dispersion Relation

Submerged Duct with Ribs

$M=1$

$M=0.1$
Pressure Directivity of a Monopole and a Dipole in a Submerged Water-filled Elastic Steel Duct

The duct has a radius $a = 1$ m and thickness $h = 0.01$ m. The unit strength monopole is located along the duct axis. The reduced frequency $\omega^* = a \omega / c_0 = 0.5$
Effect of Elastic Wall on the Blade Unsteady Lift

Impedance $\zeta=0.35i$

Impedance $\zeta=0.23i$
Summary of analysis

- Elastic wall can significantly affect the strength and location of dipole sources.

- Hydrodynamic disturbances may couple with ribbed duct modes producing dipole strength sources.

- Ribs change dispersive relation and make it possible for waves with wave length of the order of the duct radius to propagate.

- Comparison with experimental results is planned.
Conclusions

- Analytical and computational analyses yield efficient tools to model acoustically relevant fluid-structure interaction problems characterized by high frequency, complex geometry and coupling with duct modes.

- Important physical features governing these phenomena are outlined and quantified. Results can be used in engine and propeller design to reduce noise.