I. Spectral Density in White and Pink Noise

For sound in a frequency band $\Delta f$, the spectral density is defined as

$$I_f(f) = \lim_{(\Delta f) \to 0} \frac{I_b}{\Delta f},$$

where $I_b$ is the average sound intensity in a frequency band $\Delta f$ centered at $f$. This definition allows us to define the average sound intensity in a finite frequency band $\{f_1, f_2\}$ by

$$I_b = \int_{f_1}^{f_2} I_f df.$$  

1. For a one-octave band centered at 1000Hz, find the lower and upper limits $f_1$ and $f_2$, respectively.

2. A white noise is an idealized model for sound with constant spectral density. The intensity of a one-octave band of sound centered at 1000Hz is equal to 85dB. How does this intensity vary with the band center frequency $f_c$? What is the sound level of a one-octave band centered at $f_c = 250$.

3. A pink noise is an idealized model for sound with a spectral density $\propto 1/f$. If again the intensity of a one-octave band of sound centered at 1000Hz is equal to 85dB, how does this intensity vary with the band center frequency $f_c$? What is the sound level of a one-octave band centered at $f_c = 250$.

II. Sound Transmission Through a Wall

The intensity transmission coefficient for sound at a frequency $\omega$ through a wall of thickness $L = 0.1m$ separating air and water is given by
\[ T_I = \frac{4}{2 + (r_3/r_1 + r_1/r_3)\cos^2 k_2 L + (r_2^2/r_1 r_3 + r_1 r_3/r_2^2)\sin^2 k_2 L}, \] (3)

where \( k_2 = \omega/c_2 \).

1. Plot \( T_I \) versus the frequency \( f \) in Hz and particularly show what happened when \( k_2 L \approx n\pi \) and \( k_2 L \approx (n - \frac{1}{2})\pi \).

2. Estimate the narrow band of frequencies when \( k_2 L \approx n\pi \) and \( k_2 L \approx (n - \frac{1}{2})\pi \).