Homework 5

I. Consider a baffled circular piston of radius \( a \) in the horizontal plane \( x_1 - x_2 \). The piston surface \( S \) has a uniform vertical motion \( U_0 e^{-i\omega t} \). Let \( \vec{x} = \{x_1, x_2, x_3\} \) be the observation point whose spherical coordinates are \((r, \theta, \phi)\), where \( r = |\vec{x}| \), \( \theta \) is the inclination angle, and \( \phi \) is the azimuthal angle. Let \( \vec{y} = \{y_1, y_2, 0\} \) be the source point on \( S \) whose polar coordinates are \( r', \phi' \).

The expression for the pressure was derived in class and is given by

\[
p'(r, \theta, \phi) = -\frac{i\omega \rho_0 U_0}{2\pi} \int_S \frac{e^{i(k|\vec{x} - \vec{y}| - \omega t)}}{|\vec{x} - \vec{y}|} dS \tag{1}
\]

1. Develop a numerical scheme to integrate (1) numerically using spherical coordinates for the observation point and polar coordinates for the source point.

2. Test your numerical scheme by comparing the results with the closed form analytical solutions for the axial pressure, i.e., \( \theta = 0 \), and for the farfield, i.e., \( r >> a \).

3. Find and plot versus \( ka \) the smallest node angles \( \theta_1 \) for which the farfield pressure is zero.

4. If the piston is driven by a frequency such that \( \lambda = m\pi a \), Compute and plot the pressure and intensity directivities at \( r = a, r = 3a \) for \( m = 1, 2, 3, 4 \).

5. The formation of lobes at high frequency show the content of the sound spectra will be modified depending on the listener angle \( \theta \) to the axis. If the baffled oscillating piston is an approximation of a loudspeaker of radius \( a = 0.25 \text{m} \), the sound may be sharp or “edgy” along the axis and low or “dull” off the axis. Calculate the angle location and the magnitude in dB of the zero-pressure and maximum pressure of the first two nodal lobes for 100, 1000, 10,000 Hz. Comment on the results.