Homework 6

I. An air-filled duct has a rectangular cross section $x_2 - x_3$ with sides 0.5m and 0.25m, respectively. If the frequency is 1000Hz, find the propagating modes and write their expressions. Assuming that all modes have the same amplitude, plot the pressure amplitude along the duct axis of symmetry from $x_1 = 0$ to $x_1 = 10$. Also, make three-dimensional plots of the pressure at the sections $x_1 = 4$ and $x_1 = 9$.

II. A water-filled duct of square cross-section of 10 cm side.
   1. Calculate the cut-off frequencies for the lowest five modes for the following two cases: (i) rigid walls, (ii) pressure release walls.
   2. For frequencies below the lowest cut-off frequency, plot the amplitude of the evanescent waves versus the duct axial distance. Plot the distance along the duct axis of the duct at which the amplitude of the evanescent mode is reduced by $1/e$ against the frequency.
   3. If a point source were placed in the center of the cross-section, which of these modes would be excited? Is it possible to position a point source so that, of these five modes, only one is excited? if so, where?

III. An air-filled duct at standard conditions has a circular cross-section of diameter, $d = .60m$. The duct has rigid walls. A resonator at the duct inlet is radiating sound at 3000 Hz. Find the propagating modes $\{m, n\}$ and plot their acoustic pressure in the duct cross-section. Also, determine the cuton frequency for each mode.