THIN AIRFOIL THEORY

1. The basic premise of the theory is that for an airfoil in a uniform flow V_∞, the airfoil can be replaced by a vortex sheet along the chord line. The strength of the vortex sheet, $\gamma(x)$ is determined by the condition that the camber line must also be a streamline. This leads to the following *singular integral equation*

$$\frac{1}{2\pi}\oint_0^c \frac{\gamma(\xi)d\xi}{x-\xi} = V_\infty[\alpha - (\frac{dz}{dx})]. \tag{1}$$

2. It is convenient to introduce the variables θ and θ_0;

$$x = \frac{c}{2}(1 - cos\theta_0), \tag{2}$$
$$\xi = \frac{c}{2}(1 - cos\theta). \tag{3}$$

Substituting (2, 3) into (1),

$$\frac{1}{2\pi}\oint_0^\pi \frac{\gamma(\theta)sin\theta d\theta}{cos\theta - cos\theta_0} = V_\infty[\alpha - (\frac{dz}{dx})]. \tag{4}$$

Then, we assume the folowing expansion for the strength of the vortex sheet

$$\gamma(\theta) = 2V_\infty(A_0\frac{1+cos\theta}{sin\theta} + \sum_{n=1}^{\infty} A_n sinn\theta), \tag{5}$$

where $A_0, A_1, A_2, \ldots$ are constants to be determined in terms of the angle of attack α and the slope of the camber line dz/dx.

3. Substituting (5) into (4) and noting that

$$\frac{1}{\pi}\oint_0^\pi \frac{cos\, n\theta d\theta}{cos\theta - cos\theta_0} = \frac{sin\, n\theta_0}{sin\theta_0}, \tag{6}$$

gives

$$\frac{dz}{dx} = (\alpha - A_0) + \sum_{n=1}^{\infty} A_n cosn\theta. \tag{7}$$

Thus, $(\alpha - A_0), A_1, A_2, \ldots$ are the Fourier coefficients of dz/dx. Therefore, we have

$$A_0 = \alpha - \frac{1}{\pi}\int_0^\pi \frac{dz}{dx}d\theta, \tag{8}$$
$$A_n = \frac{2}{\pi}\int_0^\pi \frac{dz}{dx}cosn\theta d\theta \quad \text{for} \quad n = 1, 2, \ldots \tag{9}$$

(10)

4. All aerodynamic quantities can now be calculated from $A_0, A_1, A_2, \ldots$

$$\Gamma = \pi c V_\infty (A_0 + \frac{A_1}{2}), \tag{11}$$

$$c_l = \pi(2A_0 + A_1), \tag{12}$$

$$c_{m,le} = -\frac{\pi}{2}(A_0 + A_1 - \frac{A_2}{2}) = -\frac{c_l}{4} + \frac{\pi}{4}(A_2 - A_1), \tag{13}$$

$$x_{cp} = \frac{c}{4}[1 + \frac{\pi}{c_l}(A_1 - A_2)], \tag{14}$$

$$c_{mac} = c_{m_{c/4}} = \frac{\pi}{4}(A_2 - A_1). \tag{15}$$