Homework 4

1. We consider the flow represented by a combination of a uniform flow parallel to the x-axis,

\[ \psi_{\text{uniform}} = Uy \]

and a source

\[ \psi_{\text{source}} = \frac{Q}{2\pi}\theta, \]

where \( \theta = \tan^{-1} \frac{y}{x} \). The stream function for this combination is

\[ \psi = Uy + \frac{Q}{2\pi}\theta \tag{1} \]

(a) Calculate the velocity components \( \{u, v\} \) and show that the flow is symmetric with respect to the x-axis.

(b) Show that the stagnation point, where the two velocity components vanish, is given by \( x_s = -\frac{Q}{2\pi U}, y_s = 0 \).

(c) Show that the equation of the streamline passing by the stagnation point is

\[ Uy + \frac{Q}{2\pi}\theta = \frac{Q}{2} \tag{2} \]

(d) As \( x \to \infty, \theta \to 0 \) or \( \theta \to 2\pi \), thus the stagnation streamline has the asymptotes \( y = \pm h \) where \( h = Q/(2) \). Equation (2) can then be written as

\[ x = \frac{|y|}{\tan[\pi(1 - \frac{|y|}{h})]} \tag{3} \]

Plot the stagnation streamline for \( h = 1 \), and show that the combination of a uniform flow and a source represents a flow around a long bluff body, for example a long island in a wide river or the head of a missile.
2. Section 2.12: Problems 1 and 2.

3. Section 2.13: Problems 1, 2 and 4.

4. Section 3.2: Problems 1, and 2.

5. Section 3.3: Problems 3, and 4.

6. Section 3.4: Problems 1 and 4.