Homework 6

1. Consider a source of strength $\Lambda$ at $(-x_0, 0)$ and a sink of strength $-\Lambda$ at $(x_0, 0)$. The stream function of the combined flow is

$$\psi = -\frac{\Lambda}{2\pi} \tan^{-1} \frac{2x_0 y}{x^2 + y^2 - x_0^2}$$

(a) Give the expression of the velocity components $(u, v)$.

(b) Give the expression for $\psi$ in terms of the polar coordinates $(r, \theta)$ and calculate the velocity components $(u_r, u_\theta)$.

(c) This combination of source-sink is equivalent at distance $r >> x_0$ to a doublet of strength $\kappa = 2x_0 \Lambda$ whose stream function is given by

$$\psi = -\frac{\kappa}{2\pi} \frac{y}{x^2 + y^2} = -\frac{\kappa}{2\pi} \frac{\sin \theta}{r}$$

Calculate the velocity components $(u_r, u_\theta)$ of the doublet.

(d) As an example, let $x_0 = 0.1$, $\Lambda = 10\pi$. Compare the combination of source-sink and a doublet by plotting the variation along the lines $\theta = \pi/4, \pi/2$, and $\pi$ of $u_r$ and $u_\theta$ of the two fields versus $r$, from $r = 0.2$ to $r = 5$. Does this validate their equivalence at $r >> x_0$?

2. Section 4.4: Problems 1 and 3.
