Control Volume with One-dimensional Flux Approach

1. The x, y and z components of the material derivative of $\vec{V}$ are given by

\[
\frac{Du}{Dt} = \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z}
\]

\[
\frac{Dv}{Dt} = \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z}
\]

\[
\frac{ Dw}{Dt} = \frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z}
\]

2. For a control volume CV surrounded with a control surface CS the conservation of mass and momentum are expressed by the relations:

a. Conservation of Mass

\[
\frac{d}{dt} \int_{CV} \rho dv + \Sigma_{CS}(\dot{m})_{out} - \Sigma_{CS}(\dot{m})_{in} = 0
\]

b. Conservation of Linear Momentum

\[
\frac{d}{dt} \int_{CV} \rho u_i dv + \Sigma_{CS}(\dot{m}u_i)_{out} - \Sigma_{CS}(\dot{m}u_i)_{in} = \Sigma F_i
\]

where $\dot{m}$ is the mass flux at CS, $u_i$ is the velocity in the $i$-direction, and

\[
\Sigma F_i = \Sigma_{CS} R_i - \Sigma_{CS}[p(\vec{n}) \Delta S] + \int_{CV} \rho g_i dv
\]

$R_i$ represents the forces applied by the surroundings control surface on the fluid in the $i$-direction. Or,

\[
\Sigma F_i = -\Sigma_{CS}(F_e)_i - \Sigma_{CS}[p(\vec{n}) \Delta S] + \int_{CV} \rho g_i dv
\]

$(F_e)_i$ represents the force applied by the fluid on the body in the $i$-direction.