Inventory Competition under Fixed Order Costs

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We consider single- and multiple-period inventory competition between two firms that can replenish their inventories only at a positive fixed order cost. What ties their replenishment decisions strategically is the customers’ substitution behavior: a fixed percentage of customers facing shortages in one firm attempts to buy from the other firm. Our main contribution is to show that both multiple periods and fixed order costs, a combination of modeling elements never employed in the inventory competition literature, fundamentally impact whether there is an equilibrium. Single-period game with fixed costs always has an equilibrium, whereas the multiple-period game may not. (In contrast, equilibrium always exists in the absence of fixed order costs; this was shown in single-period (Parlar 1988) and infinite-horizon settings (Avsar and Baykal-Gursoy 2002).) We characterize the single-period equilibrium, and establish sufficient conditions for the multiple-period game to lack equilibrium. On the latter point, we offer a family of counterexamples with two periods and deterministic demands, the simplest possible multiple-period setup, strongly suggesting that the general problem with multiple periods and stochastic demands may not have an equilibrium either.

The most interesting dynamic in the multiple-period game is that a firm can improve its future sales by introducing a deliberate scarcity early on. This dynamic contributes to the nonexistence of equilibrium, and presents an idea worthwhile exploring in more general settings.

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1. Introduction

Inventory competition has been a topic of great interest in operations management. The basic story is best epitomized by the retailer maxim “stock it and they will come.” Two retailers carrying the same product at the same price would essentially compete on service dimensions, of which product availability is arguably the most important. More broadly, retailers enter into competition by stocking products that are substitutes of each other. Study of equilibrium between retailers’ inventory decisions for substitutable products is what concerns the inventory competition literature (Parlar 1988, Avsar and Baykal-Gursoy 2002, Netessine et al. 2006, Olsen and Parker 2008).

In this paper we consider two substitutable products each carried by a separate firm. The two firms replenish their inventory at a positive fixed order cost to meet random demand. For each product the demand comes from two sources: customers who prefer to shop at the firm that carries it, and customers who ordinarily prefer to shop at the other firm but need to switch loyalties due to shortage of inventory there. This substitution behavior, modeled by a fixed percentage of a firm’s excess demand switching to its competitor (an abstraction commonly employed in the literature, e.g., Parlar 1988, Avsar and Baykal-Gursoy 2002, Netessine et al. 2006), creates a strategic interaction between the firms’ inventory replenishment decisions. We study the existence and nature of equilibrium in single- and multiple-period games resulting from this interaction. (Our main point of departure from the literature is to allow non-negligible fixed order cost for each firm.)

The contribution of our paper is two-fold. We first characterize the set of single-period equilibria as a function of the firms’ initial inventory levels and fixed order costs. In particular, we observe that a pure strategy equilibrium always exists, and for some initial inventory levels multiple pure strategy equilibria may exist. We also relate the equilibrium of the general model to the equilibrium in zero-initial-inventory, zero-fixed-order-cost case (tying our results to the extant literature), and give necessary and sufficient conditions for when equilibria with both firms, one firm, and neither firm placing orders occur. Our single-period results extend the literature in the direction of incorporating fixed order costs in inventory competition. We then explore the two-period game, and establish the
nonexistence of pure-strategy subgame-perfect equilibrium through a family of counterexamples with deterministic demands. Based on the latter result, given nonexistence even in the simplest possible multiple-period setup, we conjecture that the general problem with multiple periods and stochastic demands may lack equilibrium as well. Therefore, to the extent that fixed order costs are a significant driver of inventory decisions, the inventory competition problem must be reexamined in light of our negative result for multiple-period settings.

2. Equilibrium in Single-Period Inventory Competition

In this section, we use the supermodular games framework to establish the existence of equilibrium for the single-period inventory competition model. We then characterize the nature of equilibrium. In particular, we relate the equilibrium of the general model with any set of initial inventories and fixed order costs to the equilibrium in zero-initial-inventory, zero-fixed-order-cost case. We also give necessary and sufficient conditions for when equilibria with both firms, one firm, and neither firm placing orders occur.

3. Equilibrium in Multiple-Period Inventory Competition

In this section, we show that equilibrium may not exist in the multiple-period model. To this end we present a family of deterministic models - characterized by a set of conditions - that have no equilibrium. In particular, we establish that the two-period inventory competition model does not possess an equilibrium if the following five conditions are satisfied:

1. Demands are deterministic;
2. In both periods, satisfying first-choice demand is enough to cover the fixed order cost;
3. Saving first-period inventory for the second period is worth the cost;
4. If already placing an order, it is less costly to order some additional products in the first period to meet second-period demand, than ordering those additional products in the second period at the expense of another fixed cost;
5. For at least one of the firms, if it were to place no orders in the first period, then: (a) profit potential from switchers to the other firm is not enough to cover fixed order cost for the other firm
in the second period; and (b) inventory holding cost savings that come from not having to carry inventory from the first period to the second outweigh the profits lost in the first period.

The most interesting dynamic that contributes to our nonexistence result occurs when both firms attempt to order their total two-period demand in the first period (the last item above). This strategy is not an equilibrium because each firm can profitably deviate from it by creating deliberate scarcity (by not ordering at all in the first period). The reason is subtle. Suppose firm \( i \) deviates. By not ordering in the first period, firm \( i \) would clearly give up some of its demand and lose profits. However, it can fully recover this loss in the second period. Some of the demand unmet by firm \( i \) in the first period would switch to the other firm, firm \(-i\). As a result, firm \(-i\) would have less inventory at the beginning of the second period. This would result in firm \(-i\) failing to fully meet its demand in the second period, because it would not want to place another order in the second period due to Condition (5a). Hence, some of the demand unmet by firm \(-i\) in the second period would switch to firm \( i \). Also to firm \( i \)'s advantage, by deviating it is able to avoid the holding cost associated with satisfying the second-period demand. Condition (5b) ensures that inventory holding cost savings outweigh the profits given up. In sum, the deviating firm, firm \( i \), is able to generate a strategic advantage out of scarcity that it deliberately introduces in the first period. The unfortunate byproduct of this strategic behavior is that both firms ordering their total two-period demand in the first period cannot be an equilibrium.

**References**


