Name: ______________________________

Instructor: __________________________

Math 10560, Final
May 08, 2019

- The Honor Code is in effect for this examination. All work is to be your own.
- Please turn off all cellphones and electronic devices.
- Calculators are **not** allowed.
- The exam lasts for 2 hours.
- Be sure that your name and instructor's name are on the front page of your exam.
- Be sure that you have all 17 pages of the test.

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1. (a) (b) (c) (d) (e)
2. (a) (b) (c) (d) (e)

.... 2 ....

3. (a) (b) (c) (d) (e)
4. (a) (b) (c) (d) (e)

.... 3 ....

5. (a) (b) (c) (d) (e)
6. (a) (b) (c) (d) (e)

.... 4 ....

7. (a) (b) (c) (d) (e)
8. (a) (b) (c) (d) (e)

.... 5 ....

9. (a) (b) (c) (d) (e)
10. (a) (b) (c) (d) (e)

.... 6 ....

11. (a) (b) (c) (d) (e)
12. (a) (b) (c) (d) (e)

.... 7 ....

13. (a) (b) (c) (d) (e)
14. (a) (b) (c) (d) (e)

.... 8 ....

15. (a) (b) (c) (d) (e)
16. (a) (b) (c) (d) (e)

.... 9 ....

17. (a) (b) (c) (d) (e)
18. (a) (b) (c) (d) (e)

.... 10 ....

19. (a) (b) (c) (d) (e)
20. (a) (b) (c) (d) (e)

.... 11 ....

21. (a) (b) (c) (d) (e)
22. (a) (b) (c) (d) (e)

.... 12 ....

23. (a) (b) (c) (d) (e)
24. (a) (b) (c) (d) (e)

.... 13 ....

25. (a) (b) (c) (d) (e)

Please do NOT write in this box.

Total ______________________________

You should check to see which formulas are given on the formula sheet before you begin the exam.

1.(6pts) Find the derivative of the function

$$f(x) = (2x^3 + 1)^x.$$

(Logarithmic differentiation might help.)

(a) $(2x^3 + 1)^x \left[\dfrac{6x^3}{2x^3 + 1}\right]$

(b) $\left[\ln(2x^3 + 1) + \dfrac{6x^3}{2x^3 + 1}\right]$

(c) $(2x^3 + 1)^x \left[2x^3 + 1 + \dfrac{6x^3}{2x^3 + 1}\right]$

(d) $(2x^3 + 1)^x \left[\ln(2x^3 + 1) + \dfrac{6x^3}{2x^3 + 1}\right]$

(e) $(2x^3 + 1)^x \left[\ln(2x^3 + 1) + \dfrac{1}{2x^3 + 1}\right]$

2.(6pts) Consider the following **sequences**:

$$(I)\ \left\{(-1)^n \frac{n^2 - 1}{2n^3 + 1}\right\}_{n=1}^{\infty} \qquad (II)\ \left\{e^{1/n}\right\}_{n=1}^{\infty} \qquad (III)\ \left\{\frac{n-1}{\ln(n)}\right\}_{n=1}^{\infty}$$

Which of the following statements is true?

(a) Sequence I diverges and sequences II and III converge.

(b) All three sequences diverge.

(c) Sequence III diverges and sequences I and II converge.

(d) All three sequences converge.

(e) Sequence II diverges and sequences I and III converge.

3.(6pts) Consider the following series

$$(I) \quad \sum_{n=1}^{\infty} \frac{2n^2+7}{\sqrt{n^7+2}} \qquad (II) \quad \sum_{n=2}^{\infty} \frac{2^{1/n}}{n} \qquad (III) \quad \sum_{n=1}^{\infty} \frac{\cos(n)}{n^2}$$

Which of the following statements is true?

(a) (I) diverges, (II) diverges, and (III) converges.

(b) They all diverge.

(c) They all converge.

(d) (I) converges, (II) diverges, and (III) diverges.

(e) (I) converges, (II) diverges, and (III) converges.

4.(6pts) The function $f(x) = 3x + \ln(x)$ is one-to-one (There is no need to check this). Find $(f^{-1})'(3)$.

(a) $\frac{3}{10}$ (b) 1 (c) $\frac{1}{4}$ (d) 4 (e) $\frac{1}{3}$

5.(6pts) Which of the following expressions gives the general form of the partial fraction decomposition of the function

$$f(x) = \frac{3x^4 - 4x^3 + 4x^2 - 8x + 16}{(x-2)(x^2-4)(x^2+4)}?$$

(a) $\frac{A}{x-2} + \frac{Bx+C}{x^2-4} + \frac{Dx+E}{x^2+4}$

(b) $\frac{A}{(x-2)^2} + \frac{B}{x+2} + \frac{Cx+D}{x^2+4}$

(c) $\frac{A}{x-2} + \frac{B}{x^2-4} + \frac{C}{x^2+4}$

(d) $\frac{A}{x-2} + \frac{B}{(x-2)^2} + \frac{C}{x+2} + \frac{D}{x^2+4}$

(e) $\frac{A}{x-2} + \frac{B}{(x-2)^2} + \frac{C}{x+2} + \frac{Dx+E}{x^2+4}$

6.(6pts) Evaluate $\int_0^1 x2^{(x^2)}\, dx$

(a) 1 (b) $\frac{2}{\ln(2)}$ (c) $\frac{1}{2}$ (d) $\frac{1}{\ln(2)}$ (e) $\frac{1}{2\ln(2)}$

7.(6pts) Find the sum of the following series,

$$\sum_{n=1}^{\infty} \left[\frac{n}{2^{n-1}} - \frac{n+1}{2^n} \right].$$

(a) $\frac{3}{2}$ (b) 0 (c) 1

(d) the series diverges (e) $\frac{1}{2}$

8.(6pts) Evaluate the integral

$$\int_0^{\frac{1}{3}} \sqrt{1-9x^2}\, dx.$$

Hint: The Formula sheet may help here.

(a) $\frac{\pi}{2}$ (b) 1 (c) $\frac{\pi - 1}{12}$ (d) π (e) $\frac{\pi}{12}$

9.(6pts) The solution of the initial value problem

$$xy' = y + x^2 e^x, \qquad y(1) = e + 1$$

is given by

(a) $y = x + e$ (b) $y = xe^x + x$ (c) $y = \dfrac{e^x + 1}{x}$

(d) $y = xe^x + ex$ (e) $y = xe^x + 1$

10.(6pts) Determine the following limit.

$$\lim_{x\to\infty} x\left[\cos(1/x) - 1\right] .$$

(a) ∞ (b) 1 (c) e (d) 0 (e) $-\infty$

11.(6pts) Which series below is <u>absolutely</u> convergent?

(a) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^3+1}$ (b) $\sum_{n=1}^{\infty} \frac{(-1)^n n!}{n^3}$ (c) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\sqrt{n}}$

(d) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1} \ln(n+1)}{n}$ (e) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}\pi^n}{3^n}$

12.(6pts) Use Simpson's rule with $\underline{n=4}$ to approximate the integral $\int_0^8 f(x)dx$ where a table of values for the function $f(x)$ is given below.

x	0	1	2	3	4	5	6	7	8
$f(x)$	2	1	2	3	5	3	2	1	2

<u>Note:</u> Your formula sheet may help here.

(a) 60 (b) 20 (c) 30 (d) 5 (e) 22

13.(6pts) Use your knowledge of a well known power series to find the sum of the following series

$$\sum_{n=0}^{\infty}(-1)^n\frac{\pi^{2n}}{(2n)!}.$$

(a) $\frac{1}{1-\pi^2}$ (b) 1 (c) e^{π} (d) -1 (e) 0

14.(6pts) Which line below is the tangent line to the parameterized curve

$$x = e^{\cos t} \qquad y = \sin(2t)$$

when $t = \pi/2$?

(a) $y = -x + 1$

(b) $y = 2x + 1$

(c) $y = 2x - 2$

(d) $y = x + 1$

(e) $y = -2x + 2$

 Initials: ____________

15.(6pts) Find the solution of the differential equation:

$$\frac{dy}{dx} = \frac{x^2+1}{e^y},$$

with initial condition $y(0) = 1$.

(a) $y = 1 + e^{([x^2/2]\,+\,x)}$

(b) $y = \ln\left|\frac{x^3}{3} + x + 1\right| + 1$

(c) $y = 1 + \ln\left|\ln\left|\frac{x^3}{3} + x + e\right|\right|$

(d) $y = e^{([x^3/3]\,+\,x)}$

(e) $y = \ln\left|\frac{x^3}{3} + x + e\right|$

16.(6pts) Which of the following gives a power series representation of the function

$$f(x) = \sin(2x^2)$$

centered at 0?

(a) $\sum_{n=0}^{\infty}(-1)^n 2^{4n}\,\frac{x^{4n}}{(2n)!}$

(b) $\sum_{n=0}^{\infty}(-1)^n 2^{2n}\,\frac{x^{4n}}{(2n)!}$

(c) $\sum_{n=0}^{\infty}(-1)^n 2^{2n+1}\,\frac{x^{4n+2}}{(2n+1)!}$

(d) $\sum_{n=0}^{\infty}(-1)^n 2\,\frac{x^{4n+2}}{(2n+1)!}$

(e) $\sum_{n=0}^{\infty}(-1)^n 2^{4n+2}\,\frac{x^{4n+2}}{(2n+1)!}$

17.(6pts) Use Euler's method with step size 0.2 to estimate $y(0.4)$ where $y(x)$ is the solution to the initial value problem

$$y' = 10(x-y)^2, \quad y(0) = 0.$$

(a) 2.8 (b) 0.8 (c) 0 (d) 0.4 (e) 0.08

18.(6pts) Consider the following series

$$(I)\ \sum_{n=2}^{\infty} \frac{(-1)^n}{n} \qquad (II)\ \sum_{n=2}^{\infty} \frac{n^2}{\ln(n)} \qquad (III)\ \sum_{n=1}^{\infty} \frac{3^n}{2(n!)}$$

Which of the following statements is true?

(a) Only I and II converge

(b) Only I and III converge

(c) All three diverge

(d) Only III converges

(e) All three converge

19.(6pts) Compute the radius of convergence of the power series $\sum_{n=1}^{\infty} \frac{2^n}{n^2}(x-1)^n$.

(a) $R = \frac{1}{2}$ (b) $R = \frac{3}{2}$ (c) $R = 2$

(d) $R = \infty$ (e) $R = 0$

20.(6pts) Which of the following gives the first three terms of a power series representation of the function

$$F(x) = \int \frac{1}{\sqrt{1+x^2}}\, dx$$

where $F(0) = 0$?

Hint: Your formula sheet may be helpful here

(a) $x - \frac{x^2}{6} + \frac{3x^3}{40} - \dots$ (b) $x - \frac{x^2}{6} + \frac{x^3}{40} - \dots$

(c) $x - \frac{x^3}{6} + \frac{3x^5}{40} - \dots$ (d) $x - \frac{x^3}{6} - \frac{x^5}{40} - \dots$

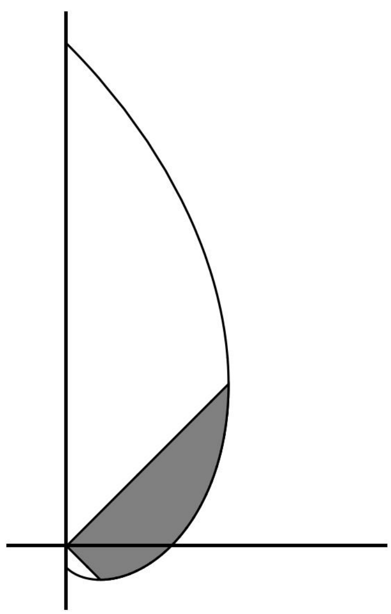
(e) $1 - \frac{x^2}{4} + \frac{3x^3}{8} - \dots$

21.(6pts) Find $\int_0^{\frac{\pi}{4}} \tan^4 x \ \sec^4 x \ dx$.

(a) $\frac{2}{15}$ (b) $\frac{1}{35}$ (c) 1 (d) $\frac{12}{35}$ (e) $\frac{2}{35}$

22.(6pts) Find the area of the region enclosed by the polar curve

$$r = e^{\theta}, \quad -\frac{\pi}{4} \leq \theta \leq \frac{\pi}{4}.$$



(a) $\frac{e^{\pi/2} - e^{-\pi/2}}{4}$ (b) $\frac{e^{\pi/4} - e^{-\pi/4}}{4}$ (c) 1/2

(d) $\frac{e^{(\pi/4)^2} - e^{(-\pi/4)^2}}{\pi}$ (e) $\frac{e^{\pi/2} - e^{-\pi/2}}{2}$

23.(6pts) Evaluate the integral

$$\int_0^{\pi/2} x^2 \cos(2x)dx.$$

(a) $\frac{\pi}{4} - 1$ (b) $-\frac{\pi}{2}$ (c) $-\frac{\pi}{4}$

(d) $\frac{\pi}{2} - 1$ (e) $\frac{1}{4} + \frac{\pi}{4} - \frac{\pi^2}{8}$

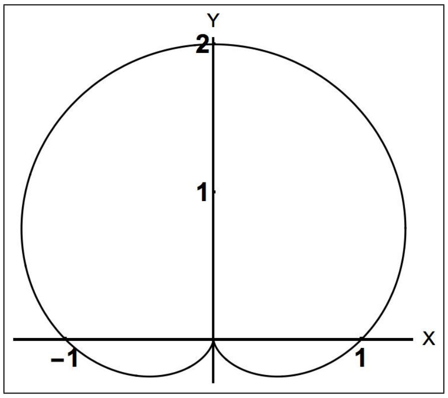
24.(6pts) Evaluate $\displaystyle\int_{1/\pi}^{\infty} \frac{\sin(1/x)}{x^2}\, dx.$

(a) $\frac{1}{2}$ (b) Divergent (c) 2

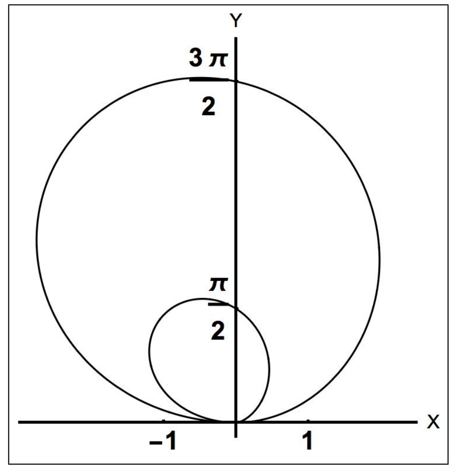
(d) $-\frac{1}{2}$ (e) 1

25.(6pts) Which graph below is the graph of the polar curve $r = \theta\cos(\theta)$ for $0 \leq \theta \leq 2\pi$?

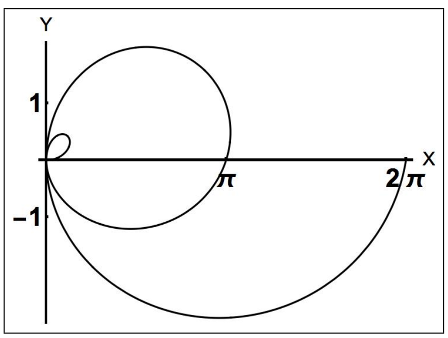
(a)



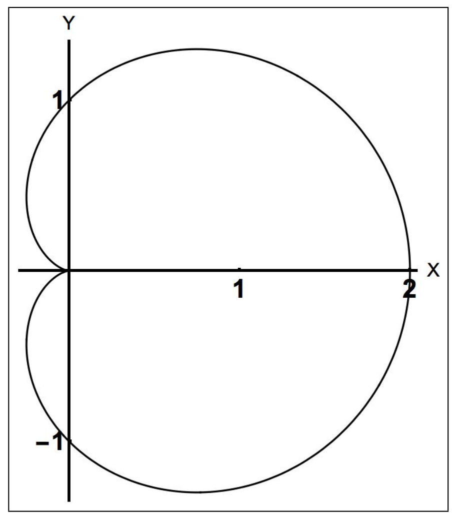
(b)



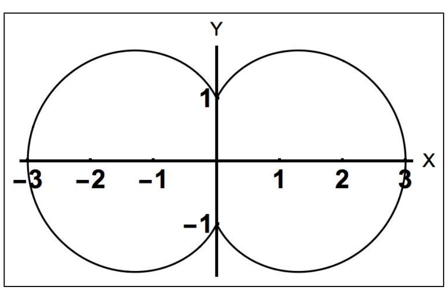
(c)



(d)



(e)



 Initials: ____________

The following is the list of useful trigonometric formulas:

$$\sin^2 x + \cos^2 x = 1$$

$$1 + \tan^2 x = \sec^2 x$$

$$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$$

$$\cos^2 x = \frac{1}{2}(1 + \cos 2x)$$

$$\sin 2x = 2 \sin x \cos x$$

$$\sin x \cos y = \frac{1}{2}\big(\sin(x - y) + \sin(x + y)\big)$$

$$\sin x \sin y = \frac{1}{2}\big(\cos(x - y) - \cos(x + y)\big)$$

$$\cos x \cos y = \frac{1}{2}\big(\cos(x - y) + \cos(x + y)\big)$$

$$\int \sec\theta d\theta = \ln|\sec\theta + \tan\theta| + C$$

$$\int \csc\theta d\theta = \ln|\csc\theta - \cot\theta| + C$$

$$\int \csc^2\theta d\theta = -\cot x + C$$

Trapezoidal Rule If f is integrable on $[a, b]$, then

$$\int_a^b f(x)dx \approx T_n = \frac{\Delta x}{2}(f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + +2f(x_{n-1}) + f(x_n))$$

Error Bounds If $|f''(x)| \leq K$ for $a \leq x \leq b$. Let E_T denote the error for the trapezoidal approximation then $|E_T| \leq \frac{K(b-a)^3}{12n^2}$

Simpson's rule If f is integrable on $[a, b]$, then

$$\int_a^b f(x)dx \approx S_n = \frac{\Delta x}{3}(f(x_0)+4f(x_1)+2f(x_2)+4f(x_3)+2f(x_4)+\cdots+2f(x_{n-2})+4f(x_{n-1})+f(x_n))$$

Error Bound for Simpson's Rule Suppose that $|f^{(4)}(x)| \leq K$ for $a \leq x \leq b$. If E_S is the error involved in using Simpson's Rule, then $|E_S| \leq \frac{K(b-a)^5}{180n^4}$

Euler's Method with step size h: $y_i = y_{i-1} + hF(x_{i-1}, y_{i-1})$, $\quad x_i = x_0 + ih$.

Binomial series If k is any real number and $|x| < 1$, then

$$(1 + x)^k = \sum_{n=0}^{\infty} \binom{k}{n} x^n$$

where $\binom{k}{n} = \dfrac{k(k-1)(k-2)\cdots(k-(n-1))}{n!}$ if $n \geq 1$, and $\binom{k}{0} = 1$.

 Initials: ____________

Rough Work

More Rough Work