ACMS 20550: Applied Math. Method I
SAMPLE EXAM

Name: ____________________________
Instructor: ________________________

- Be sure that you have all 15 pages of the test.
- You may use a BASIC calculator. (TI graphic calculators are not allowed).
- The exam lasts for 2 hours.
- Be sure that your name is on every page in case pages become detached.
- The Honor Code is in effect for this examination, including keeping your answer sheet under cover.

It is a violation of the honor code to give or to receive unauthorized aid on this Exam.

Two formula sheets are included at the end of this test

Good Luck!

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1.	a	b	c	d	e	9.	a	b	c	d	e
2.	a	b	c	d	e	10.	a	b	c	d	e
3.	a	b	c	d	e	11.	a	b	c	d	e
4.	a	b	c	d	e	12.	a	b	c	d	e
5.	a	b	c	d	e	13.	a	b	c	d	e
6.	a	b	c	d	e	14.	a	b	c	d	e
7.	a	b	c	d	e	15.	a	b	c	d	e
8.	a	b	c	d	e						

Please do NOT write below.

Multiple Choice	________
16.	________
17.	________
18.	________
Total	________

Multiple Choice

1.(6 pts.)The series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{3n+2}}$ is

(a) divergent and there is no limit for the partial sums

(b) divergent and diverges to $+\infty$

(c) divergent and diverges to $-\infty$

(d) absolutely convergent

(e) convergent but not absolutely convergent

2.(6 pts.)What value does the Fourier Series of the function

$$f(x) = \begin{cases} 3, & 0 < x < 3; \\ -1, & -3 < x < 0. \end{cases}$$

converge to at $x = 0$?

(a) -1. (b) 0. (c) 1.

(d) 2. (e) 3.

3.(6 pts.)If the temperature is $T = x^2 - xy + z^2$, the rate of change of temperature in the direction $\mathbf{j} - \mathbf{k}$ at $(2, 1, -2)$ is

(a) 0, (b) $\sqrt{2}$, (c) 1,

(d) -1, (e) $-\sqrt{2}$.

4.(6 pts.) The power series $\sum_{n=1}^{\infty} \frac{2n+1}{2^n} x^n$ is

(a) convergent for $|x| < 2$ and divergent for $|x| > 2$

(b) convergent for $|x| > 3$ and divergent for $|x| < 3$

(c) convergent for $|x| < 3$ and divergent for $|x| > 3$

(d) convergent for $|x| > 2$ and divergent for $|x| < 2$

(e) convergent for $|x| < 1$ and divergent for $|x| > 1$

5.(6 pts.)The series $\sum_{n=1}^{\infty} \frac{n^2+5}{\sqrt{n^5+2n^2+6n}}$ is

(a) convergent by ratio test

(b) divergent by ratio test

(c) divergent by integral test

(d) convergent by comparison test

(e) divergent by comparison test

6.(6 pts.) The series $\sum_{n=1}^{\infty} \left(\frac{\sqrt{2}+i}{1+i\sqrt{3}}\right)^n$ is

(a) convergent by the integral test

(b) divergent by the comparison test.

(c) divergent by preliminary test.

(d) convergent by ratio test.

(e) none of the above.

7.(6 pts.)Find the tangent plane of the surface $x^3y^2z = 2$ at the point $(1, -2, 3)$. The tangent plane is

(a) $2(x-1) + 2(y+2) + (z-3) = 0.$

(b) $9(x-1) - 3(y+2) + 2(z-3) = 0.$

(c) $2(x-1) + (y+2) + (z-3) = 0.$

(d) $9(x-1) - 3(y+2) + (z-3) = 0.$

(e) $3(x-1) + 3(y+2) + (z-3) = 0.$

8.(6 pts.)For which value of a is the vector field $\mathbf{F} = [a\,y^2 + x]\,\mathbf{i} + [y - xy]\,\mathbf{j}$, conservative?

(a) -1

(b) $-1/2$

(c) 0

(d) 1

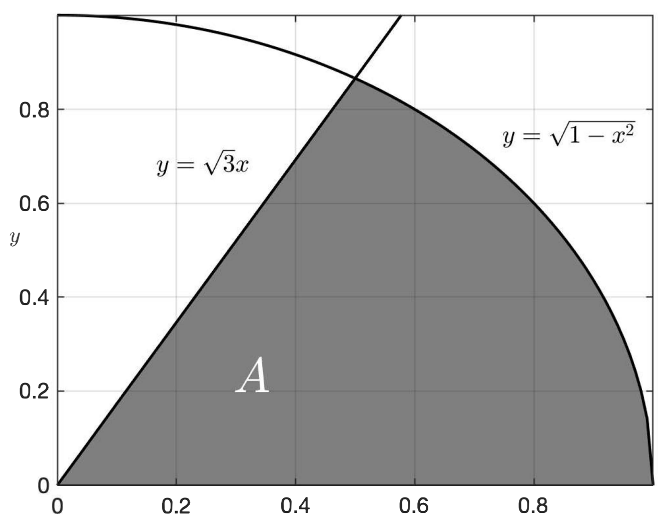
(e) -2

9.(6 pts.) If $x^2 + y^2 = 2st - 10$ and $2xy = s^2 - t^2$, then $\dfrac{\partial x}{\partial t}$ is

(a) $\dfrac{sx + ty}{x - y}$ (b) $\dfrac{sx + ty}{x + y}$ (c) $\dfrac{sx - ty}{x + y}$

(d) $\dfrac{sx + ty}{x^2 + y^2}$ (e) $\dfrac{sx + ty}{x^2 - y^2}$

10.(6 pts.) For the area A in the picture below bounded by the x axis, the line $y = \sqrt{3}x$ and the portion of the circle $x^2 + y^2 = 1$. Which of the following represents the integral $\iint_A f(x, y)dxdy$.



(a) $\displaystyle\int_{x=0}^{1}\left(\int_{y=0}^{\sqrt{1-x^2}} f(x, y)dy\right) dx.$ (b) $\displaystyle\int_{y=0}^{1/2}\left(\int_{x=y/\sqrt{3}}^{\sqrt{1-y^2}} f(x, y)dx\right) dy.$

(c) $\displaystyle\int_{x=0}^{1/2}\left(\int_{y=0}^{\sqrt{3}x} f(x, y)dy\right) dx.$ (d) $\displaystyle\int_{y=0}^{\sqrt{3}/2}\left(\int_{x=y/\sqrt{3}}^{\sqrt{1-y^2}} f(x, y)dx\right) dy.$

(e) $\displaystyle\int_{x=0}^{1}\left(\int_{y=\sqrt{3}x}^{\sqrt{1-x^2}} f(x, y)dy\right) dx.$

11.(6 pts.) The value of $\int_C \mathbf{F} \cdot d\mathbf{r}$ where C is arc of the the circle $x^2 + y^2 = 2$ from $(1,1)$ to $(1,-1)$ and $\mathbf{F} = (2x - 3y)\,\mathbf{i} - (3x - 2y)\,\mathbf{j}$, is

(a) -3 (b) 0 (c) 6

(d) -6 (e) 3

12.(6 pts.) For large n, the expression $\dfrac{1}{n^3+1} - \dfrac{1}{n^3}$ can be approximated by

(a) $\dfrac{3}{n^6}$. (b) $-\dfrac{1}{3n^6}$. (c) $-\dfrac{1}{n^6}$.

(d) $-\dfrac{3}{n^6}$. (e) $\dfrac{1}{n^6}$.

13.(6 pts.)The value of $\lim_{x\to 0^+} \dfrac{\sin x^2}{x(1-\cos\sqrt{x})}$ is

(a) -2. (b) -1. (c) 1.

(d) 2. (e) 0.

14.(6 pts.) The values of $(-1)^i$ for $n = 0, \pm 1, \pm 2, \ldots$ are

(a) $e^{2\pi-\pi n}$. (b) $e^{-\frac{\pi}{2}+2\pi n}$. (c) $e^{\pi-2\pi n}$.

(d) $e^{-2\pi-2\pi n}$. (e) $e^{\frac{\pi}{3}+2\pi n}$.

15.(6 pts.) Which of the following represents the integral

$$\int_{-1}^{1}\left(\int_{0}^{\sqrt{1-x^2}} f(\sqrt{x^2+y^2})\mathrm{d}y\right)\mathrm{d}x$$

(a) $2\pi\int_0^1 f(r)r\mathrm{d}r$

(b) $\frac{\pi}{2}\int_0^1 f(r)\mathrm{d}r$

(c) $2\pi\int_0^1 f(r^2)\mathrm{d}r$

(d) $\frac{\pi}{2}\int_0^1 f(r)r\mathrm{d}r$

(e) $\pi\int_0^1 f(r)r\mathrm{d}r$

Partial Credit

You must show your work on the partial credit problems to receive credit!

16.(20 pts.) (a) Calculate the surface area of the surface $2z = x^2+y^2$ over the disc $x^2+y^2 \leq 9$ and above the xy plane.

(b) Evaluate $\displaystyle\oiint_S (2z^2x\mathbf{i} + z\mathbf{j} - y\mathbf{k}) \cdot \mathbf{n}\, \mathrm{d}S$ where S is the closed surface of the cylinder $x^2 + y^2 \leq 4$ which lies between the planes $z = 0$ and $z = 1$.

17.(20 pts.) Use Lagrange multipliers to find the dimensions of the largest cuboid which can fit inside an ellipsoid, i.e. maximize $V = xyz$ subject to $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} + \dfrac{z^2}{c^2} = 1$ for constants a, b, c.

18.(20 pts.) Compute the Fourier Cosine Series of

$$f(x) = \begin{cases} \pi - x, & 0 < x < \pi, \\ \pi + x, & -\pi < x < 0. \end{cases}$$

Formula Sheet

1. Taylor Series:

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + x^4 + \cdots, -1 < x < 1;$$

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots, \text{ all } x;$$

$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots, \text{ all } x;$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots, \text{ all } x;$$

$$\ln(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^n}{n} = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots, -1 < x \leq 1;$$

2. Tangent Plane and Normal Line:

If the normal vector direction is given by $\vec{n} = (n_1, n_2, n_3)$, then

Tangent plane at (x_0, y_0, z_0) is: $\quad n_1(x - x_0) + n_2(y - y_0) + n_3(z - z_0) = 0$

Normal line at (x_0, y_0, z_0) is: $\quad \dfrac{x - x_0}{n_1} = \dfrac{y - y_0}{n_2} = \dfrac{z - z_0}{n_3}$

Formula Sheet

3. Surface integral on D represented by $\phi(x, y, z) = 0$

$$\iint_D dA = \iint \sec\gamma dxdy, \quad \sec\gamma = \frac{\sqrt{\left(\frac{\partial\phi}{\partial x}\right)^2 + \left(\frac{\partial\phi}{\partial y}\right)^2 + \left(\frac{\partial\phi}{\partial z}\right)^2}}{\left|\frac{\partial\phi}{\partial z}\right|}$$

Surface integral on D represented by $z = f(x, y)$

$$\iint_D dA = \iint \sec\gamma dxdy, \quad \sec\gamma = \sqrt{1 + \left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$$

4. Green's Theorem: For bounded region A and continuous $P(x, y)$, $Q(x, y)$

$$\iint_A \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dxdy = \oint_{\partial A} (Pdx + Qdy).$$

5. Stokes' Theorem: For a surface σ in 3-dimensional space with its boundary $\partial\sigma$ oriented in consistency with the normal direction,

$$\oint_{\partial\sigma} \mathbf{V} \cdot d\mathbf{r} = \iint_\sigma \text{curl } \mathbf{V} \cdot \mathbf{n} \; d\,\sigma.$$

6. Volume generated by the vectors $\vec{A}, \vec{B}, \vec{C}$.

If $\vec{A} = (A_1, A_2, A_3)$, $\vec{B} = (B_1, B_2, b_3)$, $\vec{C} = (C_1, C_2, C_3)$, then

$$\vec{A} \cdot (\vec{B} \times \vec{C}) = \begin{vmatrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \\ C_1 & C_2 & C_3 \end{vmatrix}$$

7. Polar coordinate system $(r \geq 0,\ 0 \leq \theta \leq 2\pi)$

$$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases}, \quad dxdy = rdrd\theta$$

8. Cylindrical coordinate system $(r \geq 0,\ 0 \leq \theta \leq 2\pi)$

$$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \\ z = z \end{cases}, \quad dxdydz = rdrd\theta dz$$

9. Spherical coordinate system $(r \geq 0,\ 0 \leq \theta \leq \pi,\ 0 \leq \phi \leq 2\pi)$

$$\begin{cases} x = r\sin\theta\cos\phi \\ y = r\sin\theta\sin\phi \\ z = r\cos\theta \end{cases}, \quad dxdydz = r^2\sin\theta drd\theta d\phi$$