

Summary of Convergence and Divergence Tests

Test	Series	Convergence/Divergence	Comments
n th term	$\sum a_n$	Diverges if $\lim_{n \rightarrow \infty} a_n \neq 0$	If $\lim_{n \rightarrow \infty} a_n = 0$, need another test
Geometric Series	$\sum_{n=1}^{\infty} ar^{n-1}$	Diverges if $ r \geq 1$ Converges if $ r < 1$	If $ r < 1$, converges to $S = \frac{a}{1-r}$
p -series	$\sum_{n=1}^{\infty} \frac{1}{n^p}$	Diverges if $p \leq 1$ Converges if $p > 1$	Harmonic Series when $p = 1$
Integral Test	$\sum_{n=1}^{\infty} a_n$	Diverges if $\int_1^{\infty} a_n \, dn$ diverges Converges if $\int_1^{\infty} a_n \, dn$ converges	a_n must be positive, continuous, and decreasing
Comparison Test	$\sum a_n, \sum b_n$ $a_n > 0$ and $b_n > 0$	If $\sum b_n$ converges and $a_n \leq b_n$ for all n , then $\sum a_n$ converges If $\sum b_n$ diverges and $a_n \geq b_n$ for all n , then $\sum a_n$ diverges	$\sum b_n$ is often a p -series or geometric series
Special (Limit) Comparison Test	$\sum a_n, \sum b_n$ $a_n > 0$ and $b_n > 0$	If $\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = L > 0$, then both converge or both diverge	L is a finite number
Alternating Series	$\sum_{n=1}^{\infty} (-1)^n a_n$ $a_n > 0$	Converges if $\lim_{n \rightarrow \infty} a_n = 0$ and $a_{n+1} \leq a_n$	Terms must alternate!
Absolute Convergence	$\sum a_n$	If $\sum_{n=1}^{\infty} a_n $ converges, then $\sum_{n=1}^{\infty} a_n$ converges If $\sum_{n=1}^{\infty} a_n $ diverges, then we need another test for $\sum_{n=1}^{\infty} a_n$	Absolute convergence implies convergence
Ratio Test	$\sum a_n$	If $\lim_{n \rightarrow \infty} \left \frac{a_{n+1}}{a_n} \right = L$ or ∞ , then Diverges if $L > 1$ or ∞ Converges if $L < 1$ Inconclusive if $L = 1$	Useful if a_n involves factorials or n th powers